

① FUNDAMENTAL OF SURVEYING.

Surveying is art of determining relative positions of points on, above and below the earth surface presenting it Graphically or numerically.

Objectives of surveying:-

- 1) To determine relative position of point
- 2) To layout or markout proposed structure on the ground
- 3) To Determine relative quantities like Area & Volume

METHODS of presentation:-

① Numerical representation:- $AB = 10\text{KM}$

② Graphical representation:- $A \xrightarrow{10\text{KM}} B$

Graphical representation is done in the form of plan or map.

plan:- $\rightarrow$ large scale

map:- small scale

NOTE:- Vertical distance can be presented with the help of contour or spot level.

contour is a imaginary line joining the points of same elevation on the earth surface

- spot levels are height of individual points
- Contour gives better visualization in comparison to spot level

BASIC DEFINITION :-

① shape of earth :-

a) oblate spheroid → slightly flat at poles. polar axis is 43.5 km smaller than equatorial axis

b) elipsoide → equatorial section is elliptical in nature

c) ovaloid → Southern hemisphere is slightly larger than northern hemisphere.

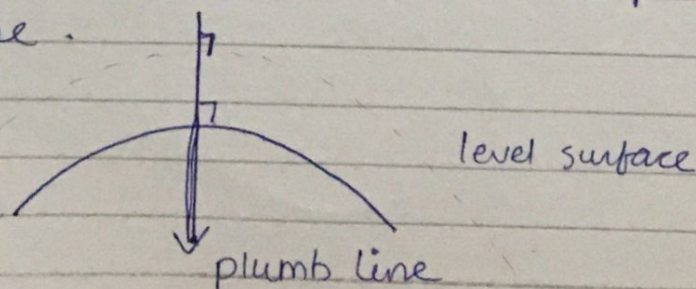
We can observe that no geometrical figure completely defines shape of earth. Hence, a new name is given to the shape of earth that is 'Geoid'

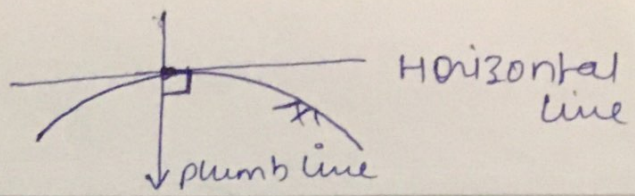
NOTE :- For calculation purpose we'll assume earth to be spherical in nature.

② level surface :-

Level surface is a curved surface parallel to the spherical surface of earth and hence every point on it is equidistant from the centre of the earth.

every element on level surface is perpendicular to plumb line.





③ Horizontal plane :-

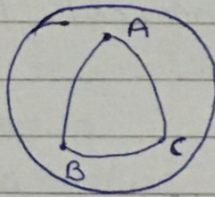
It is a tangential plane to the earth surface which is also perpendicular to plumb line.

④ Great circle :- It is a imaginary circle passing through the centre of the earth.

A Great circle devides earth surface into two equal parts.

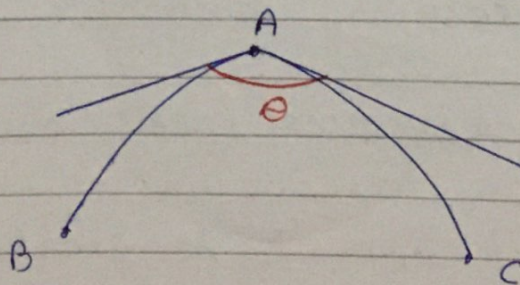
⑤ spherical triangle :-

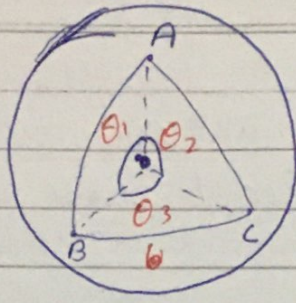
Spherical triangle is a triangle which is formed on the surface of a sphere by intersecting by 3 arcs of different Great circle.



The arc enclosing spherical triangle are called as its side and the angles in which these arc intersect are called as spherical angles.

Spherical angle is defined as angle b/w the tangent to the great circles and the points of intersection





$$\widehat{AB} = R\theta_1$$

$$\widehat{BC} = R\theta_3$$

$$\widehat{AC} = R\theta_2$$

Length of a side of a spherical triangle is defined as angle subtended by that side at the center of the sphere

properties of spherical triangle :-

- each side of a spherical triangle must be less than equal to ' π '
- each angle of a spherical triangle must be less than ' 180° '
- sum of 3 sides must be b/w ' $0 - 360^\circ$ '
- sum of 3 angles of a spherical triangle must be b/w ' $180^\circ - 540^\circ$ '

$\downarrow$
inclusive

$\rightarrow$ Exclusive

NOTE Amount by which sum of angles of a spherical triangle exceed by 180° is called as spherical excess

- Surface Area of a spherical triangle must be less than $2\pi r^2$
- Greater angle is opposite to greater side.

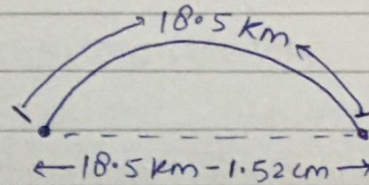
Classification of surveying :-

Surveying can be classified into various means

Ex: instrument used, purpose, place of survey etc.

But mainly surveying is classified into two types.

- 1) Geodetic survey
- 2) plane survey



Geodetic

- It is done for large Area and we consider earth surface as curved.
- Area $\geq 195.5 \text{ km}^2$
- It is done by survey of India to establish control points throughout the country
- It uses spherical trigonometry

plane survey

- plane surveying is done for small area and we consider earth surface as flat surface that is curvature of earth is neglected
- Area $< 195.5 \text{ km}^2$
- It is done for local survey
- It uses plane trigonometry

NOTE:- In plane surveying plotted measurements are projected on a horizontal plane

For Area = 195.5 km^2 spherical excess = 1 second

Classification of surveying based on purpose of survey

1. Topographical Survey:- These survey are use to obtain maps which show details of Natural and Manmade features, on the earth surface including elevation information.

Scale:- 1:25000 to 1:1000000

2. Engineering surveying:- These survey are used for engineering works

Ex' Dam, Highway, Canal, railway etc.

Building work $\rightarrow$ 1:50 to 1:200

Bridges and other civil engineering work $\rightarrow$ 1:500 to 1:2500

Highway $\rightarrow$ 1:1250 to 1:50000

3. CADASTRAL Surveying:- These are used to represent property boundaries

Scale $\rightarrow$ 1:1000 to 1:5000

4. Hydrographic Surveying:- These survey are done on or near the water body such as lake, river, bay etc.

5. Astronomic survey:- With the help of Astronomic Survey we can determine latitude, longitude and local mean time of any place

6. Geological survey:- It is used to obtain info. about different strata of earth surface.

classification based on instrument :-

① chain survey :-

it is simplest type of surveying in which only linear measurements are done with the help of chain or tape.
• No angular measurement is done

② compass survey :-

In compass surveying directions and horizontal angles are measured with the help of compass and distances are measured with the help of chain or tape.

③ levelling :-

In this type of surveying vertical distances are measured with respect to a fixed frame of reference which is generally called as 'Datum'

④ plane table surveying :-

In plane table surveying plan or map is prepared by determining direction of various points and taking their linear measurements with chain or tape

⑤ Theodolite surveying :-

In this kind of surveying horizontal & vertical angles are measured with the help of theodolite and distances are measured with the help of chain or tape

⑥ Tachometric surveying :-

In this kind of surveying we measure horizontal & vertical distances optically with the help of instrument called as tachometer

⑦ photogrammetric surveying:-

In this kind of surveying we survey a Area with the help of photographs.

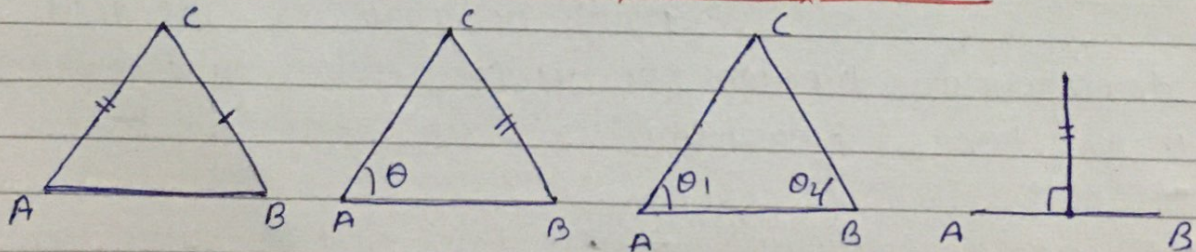
Principles of surveying:-

1) Working from whole to part:-

Main objective of working from whole to part is to localise the error and prevent their accumulation.

If we work from part to whole error gets accumulation or it gets maximised to greater extend.

2) Locate a point atleast by two measurements:-



SCALE:-
$$\text{Scale} = \frac{\text{length of a line on plan or Map}}{\text{Length of Same Line on the ground}}$$

Scale

Numerical scale

Graphical scale

Engineer scale

1 cm of plan or
Map distance
= ground distance

Representative

fraction (Rf)

it is Fractional
form of scale

1:10⁶

large scale & small scale :-

SCALE A

$$1 \text{ cm} = 10 \text{ m}$$

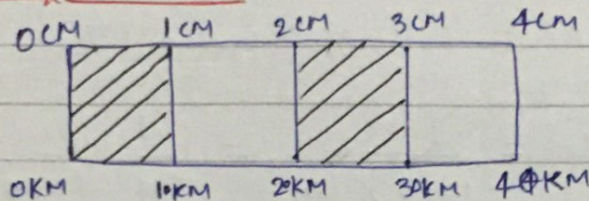
SCALE B

$$1 \text{ cm} = 100 \text{ m}$$

larger the denominator smaller will be the scale.

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Graphical scale :-



Graphical scale is a line drawn on map and marking ground distance directly on it.

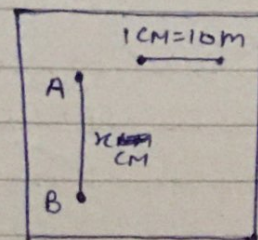
Graphical scale has advantage over numerical scale because distance on the map can be determined by actual scaling even though map has shrunk.

in case of shrinkage of map graphical scale also changes with the map Hence, ratio is unaffected.

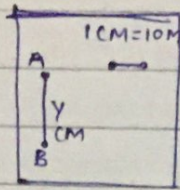
i) before shrinkage :-

- Let distance b/w two points on the plan or map equals to x cm
- let scale is 1:1000
- Therefore x cm on plan or map equals to a ground distance of $1000x$ cm.

— After shrinkage



ii) AFTER Shrinkage :-



- Now the distance b/w same points on plan or Map equals to y cm.
- y cm on plan or Map = $1000 \times$ cm of ground distance

$$\text{Shrunk scale} = \frac{y \text{ cm}}{1000 \times \text{cm}} = \frac{y}{x} \times \frac{1}{1000} \quad \text{original scale}$$

Shrinkage factor or shrinkage ratio equals to

$$\frac{\text{shrink length (y)}}{\text{original length (x)}} = \frac{\text{shrunk scale}}{\text{original scale}}$$

$$= \frac{\text{shrunk R.F}}{\text{original R.F}}$$

- shrinkage factor is always < 1

- Shrunk scale $\rightarrow$ small scale
- original scale $\rightarrow$ large scale

Que. plan of an Area has shrunk such that A line originally 10 cm long now measures as 9.5 cm only. if original scale of the plan was 1 cm = 10 m (R.F = 1:1000) then determine -

- shrinkage factor
- shrunk scale
- correct length if measured length of line was 100 m and correct Area if measured Area was 100 m²

Solⁿ

$$a) S.F = \frac{9.5}{10} = 0.95$$

$$b) S.S = 0.5 \times S.F \\ = \frac{1}{1000} \times 0.95$$

$$S.S = \frac{1}{1052.63}$$

$$c) S.F = \frac{y}{x} \times \frac{1000}{1000} = \frac{1000y}{1000x}$$

$$S.F = \frac{\text{Measured distance}}{\text{Correct distance}}$$

$$\text{Correct Distance} = \frac{\text{Measured length}}{\text{shrinkage factor}}$$

$$C.D = \frac{100}{0.95} = 105.263 \text{ m}$$

$$C.A = \frac{100}{(0.95)^2} = 110.803 \text{ m}^2$$

Error Due to Wrong Measuring scale :-

$$EL = \frac{\text{R.F of Wrong scale}}{\text{R.F of correct scale}} \times M.L$$

Que. A surveyer measures the distance b/w two points marked on the plan drawn to a scale of $1\text{ cm} = 1\text{ m}$, and found it to be 50 m . Later he discovered that he has used a wrong scale of $1\text{ cm} = 50\text{ cm}$ for the measurement determine correct length, what would be correct Area if measured Area is 100 m^2 .

$$C.L = \frac{1/50}{1/100} \times 50 = 100\text{ m.}$$

$$C.A = \left(\frac{1/50}{1/100} \right)^2 \times 100 = 400\text{ m}^2$$

USE OF VERNIER in SCALE:-

Vernier is a device used for measuring the reading which are fractional part of smallest division on main scale

The reading is given by

Reading before index mark on main scale + $N \times L.C$

$N \rightarrow$ number of vernier division which perfectly coincides with main scale division

$L.C \rightarrow$ one division of main scale - one division of vernier scale.

Types of vernier:-

(i) Direct vernier:- n division of vernier scale are equal to $(n-1)$ division of main scale one vernier division is slightly smaller than one division of main scale

$$n(V) = (n-1)S$$

$$L.S = S - V$$

$$L.S = S - \left(\frac{n-1}{n}\right)S$$

$$L.C = \frac{S}{n}$$

2) Retrograde vernier :-

- n divisions on vernier scale are equal to $(n+1)$ division on main scale
- one vernier division is slightly larger than one division on main scale
- vernier division and main scale division increases in opposite dirⁿ whereas in direct vernier both increases in same direction

$$nV = (n+1)S$$

$$L.C = V - S$$

$$L.C = \left(\frac{n+1}{n}\right)S - S$$

$$L.C = \frac{S}{n}$$

Que · The main plate of a theodolite is divided into 1080 equal parts and 60th division of vernier scale coincides with 59th division of main scale. determine least count of theodolite?

$$L.S = \frac{S}{n} = \frac{(360^\circ/1080)}{60}$$

$$= 20''$$

Special types of vernier :-

1) Double vernier :-

if graduation of main scale increases in one dir only we have a single vernier whereas main scale is graduated in both the directions a double vernier is used

Ex: Theodolite

Double vernier consist of two direct verniers extending in both the direction with index mark in the centre.

2) Double folded vernier :- A double folded vernier is a special type of double vernier whose total length is exactly half of combined length of corresponding double vernier

- A double folded vernier is used when it is required to reduce the length of vernier scale.

Ex: sextant

- in double folded vernier the vernier is read from index mark (0 mark) towards either of the extream deviation and then from other extream deviation towards centre in the same direction.

3) EXTENDED VERNIER ⇒

When divisions of main scale are very close and it becomes very difficult to determine exact graduation where coincidence occurs if vernier of normal length is used Hence Extended vernier is very useful when available length of vernier scale is small and it is required to have small least count without making vernier deviation very close

- In extended vernier n division of vernier scale are equal so $2n-1$ division of main scale

$$nV = (2n-1)S$$

$$L.C = 2S - \left(\frac{2n-1}{n}\right)S$$

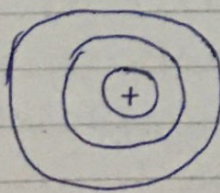
$$L.C = \frac{S}{n}$$

Precision and accuracy:-

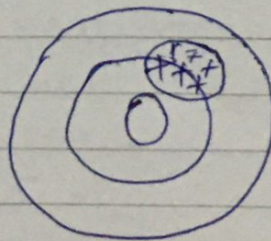
precision:- Degree of fineness and care with which any physical measurement is done

Accuracy:- Degree of perfection

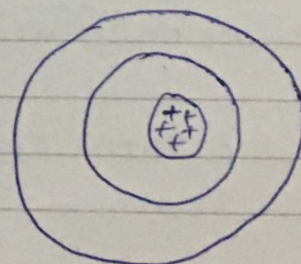
- A value is considered to be accurate when it is close to true value
- precise value Represents set of observations that are closely grouped and have small deviation from true value



A.V



P.V



A.V & P.V

NOTE To plot or draw a plan accuracy required is also defined by scale of the plan. A good

draftman can plot smallest length of 0.25 mm, Now for a scale of 1:1000 (that is 1 mm on plan = 1 m on ground), smallest plotable distance is 0.25 m. Hence For survey at scale of 1:1000 Accuracy desired in the field should be 0.25 m

Mistake and Error:-

Mistake:- These are the discrepancies caused due to carelessness, misunderstanding and poor judgement.

- Mistakes can be corrected if only discovered, mistakes can be avoided by adopting careful procedure

Error:- These are the discrepancies other than mistake.

$$\text{Error} = M \cdot V - T \cdot V$$

Error = - correction

therefore correction = $T \cdot V - M \cdot V$

Error

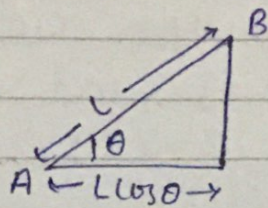
Systematic error
or
cumulative error

- These errors follow a definite mathematical or physical law
- These are cumulative in nature

Random error
or
compensating error
or
accidental error

- These are the errors other than systematic error
- These are compensating in nature

[2] LINEAR MEASUREMENT



- Main purpose of surveying is to plot map of an Area
- As The map is plotted on a horizontal plane. The distances shown are horizontal projection on this plane.
- In surveying the distance b/w 2 points is Horizontal distance
- When the slop distances are measured in the field these are always reduced to equivalent horizontal distance for the preparation of Map.

Methods of Linear/Horizontal Measurement:-

- ① Direct Method :- chain or tape
- ② optical Method :- Tacheometry
- ③ Electronic Distance Measuring instrument (EDMI)
Geodimeter, Distomat
Mekometer, Tellurometer
Total station*

CHAIN Surveying:-

- In this type of surveying the distance are measured with chain or tape.

- chain surveying is used for smaller Area

- In the process of chaining the survey team consist of follower (0 m end) and leader (20 or 30 m end)
- leader puts the arrow in the ground and follower collects it
- Number of Arrows with the follower signifies chain length measured

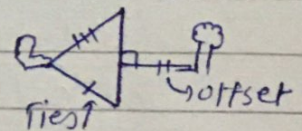
procedure in chain surveying :-

for small Area details of various points can be collected with the help of ties or offsets.

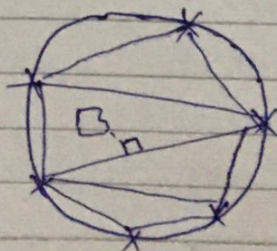
NOTE

Offsets are lateral distances measured from the survey line to the point whose information to be collected.

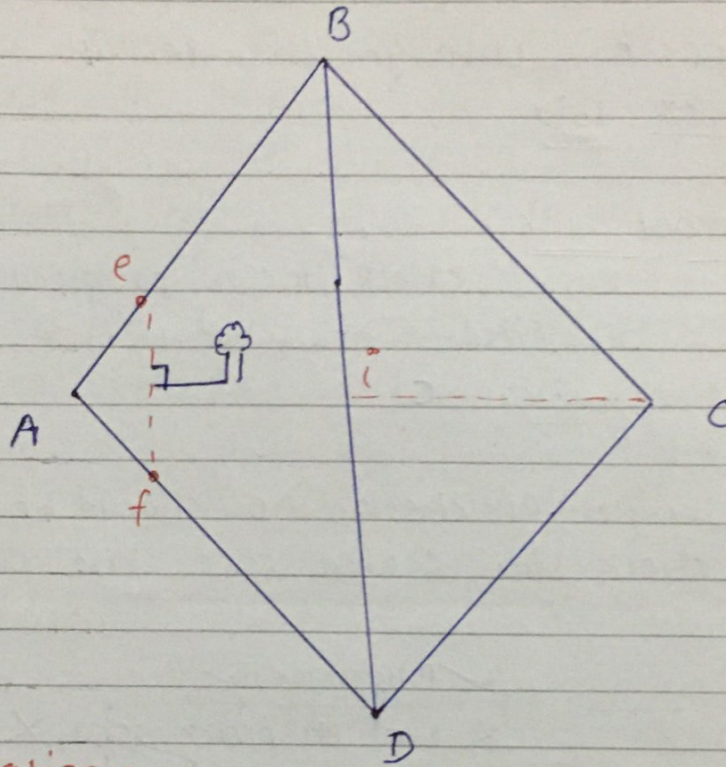
perpendicular offset $\rightarrow$ simply offset
oblique offset $\rightarrow$ Tie



- When Area is large Error introduced due to large offset will be more. Therefore to overcome this difficulty large Area is first at all converted into a network of triangle and then details are collected with respect to sides of these triangle using ties and offset



BASIC DEFINITIONS :-



Main station :-

Main station is a point where two sides of a triangle meet and these station decides boundary of the survey area

Ex: A, B, C & D

Main survey line :-

These are the lines joining main stations Ex: AB, BC, CD, DA, BD

Tie station / subsidiary station / ~~Auxiliary~~ Auxiliary station :-

These are the station on main survey line Ex: e & f

Tie line / Auxiliary line / subsidiary line :-

These are the lines joining tie stations.

- These are helpful in locating inner details of the Area which is to be surveyed Ex: ef

Base Line:-

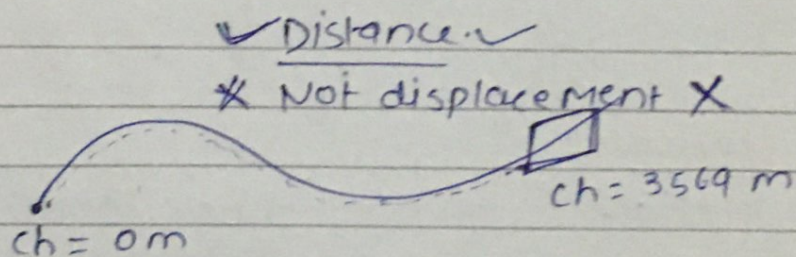
It is longest main survey line which generally passes through the central part of the Area. Ex: BD

check line/ proof line :-

check line is provided to check the accuracy of field work, it is **NOT** for plotting of any point Ex: CI

- Measured length of check line should be equal to length of check line scaled from the plan

Chainage :-



- chainage is the distance measured along the dirⁿ of the progress of survey
- It is distance of a well defined point from the starting point
- The term chainage is generally used in linear projects Ex: highway, railway, canal etc.
- They are used to represent important features in the work.

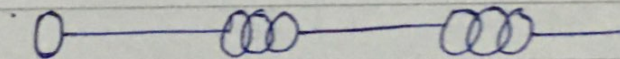
Well conditioned triangle :-

In chain surveying attempt should be made to form a triangle such that intersections of lines should be clear for plotting purpose. Hence, for obtaining clear intersection angles should be in the range of $30^\circ - 120^\circ$.

- Best possible Well conditioned triangle is equilateral triangle.

Equipments used in chain surveying :-

1) Surveying chain :-



Types of chains :-

- 1) Revenue chain $\rightarrow$ 33 ft 16 Links
- 2) Gunter's chain $\rightarrow$ 66 ft 100 Links
- 3) Engineer's chain $\rightarrow$ 100 ft 100 Links
- 4) Metric chain $\rightarrow$ a) 20 m $\rightarrow$ 100 link
b) 30 m $\rightarrow$ 150 Link

NOTE :- 1 mile = 1.6093 Km = 80 Gunter's chain
1 acre = 4046.856 m² = 10 $\times$ (Gunter's chain)²
1 yard = 3 ft = 0.9144 m

2) TAPE :-

① Cloth or linen tape :- It is made up of closely woven linen or

synthetic material and varnished to resist moisture.

- This tape doesn't remain straight during strong winds.

② Metallic tape:-

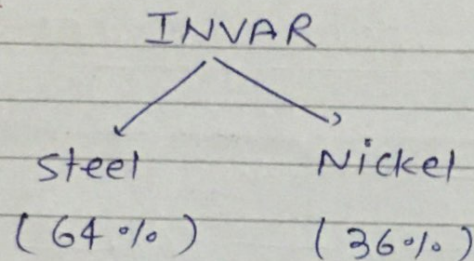
It is made up of linen with brass or copper wire woven into it and varnished.
• This tape gives better result than in comparison to cloth tape because stretching is reduced

③ Stainless Steel tape:-

Steel

It is made up of stainless

④ INVAR tape:-



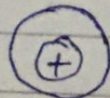
These tapes are highly accurate and have small value of thermal coefficient of linear expansion (α)

• This tape is very soft and hence required special attention on the field.

Accessories used in Surveying:-

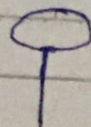
① pegs:-

These are used to mark definite point on the ground semi permanently or temporarily.



② Arrow:-

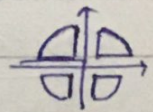
It is used by leader to mark end point of the chain length which is later collected by follower



③ Ranging Rod :- It is used to locate intermediate points along a straight line and this process is called as Ranging.

④ Offset Abstract Rod :-

It is a ranging rod with slots made at right angle, it can be used to establish perpendicular offset with respect to survey line.

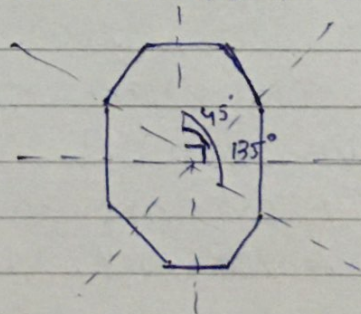


⑤ Cross staff / open cross staff :-

It is used for establishing perpendicular offsets with respect to survey line at any point.

⑥ French cross staff :-

With the help of French cross staff we can establish 45° , 90° & 135° offsets.



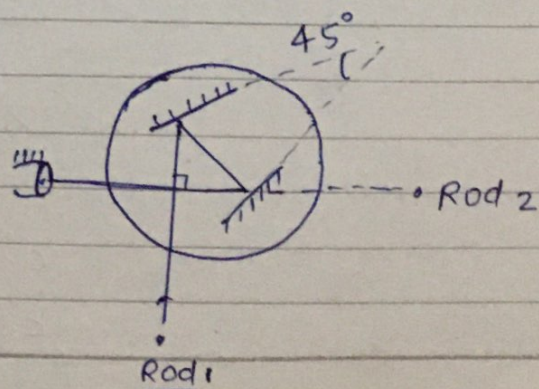
⑦ Optical square :-

• It is used to set out perpendicular offsets.

• It used principle of double reflection.

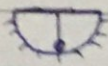
• Index mirror fully silvered.

• Horizon mirror $\rightarrow$ half silvered and other half not silvered.



⑧ prism square:- It has similar arrangement as compare to optical square with mirror replaced by prism.

⑨ Simple clinometer:- It is used to measure slope of the ground or vertical angle



Ranging:- It is the process of establishing intermediate points along a straight line

① Direct Ranging:-

When intermediate points are established by direct observation from end stations

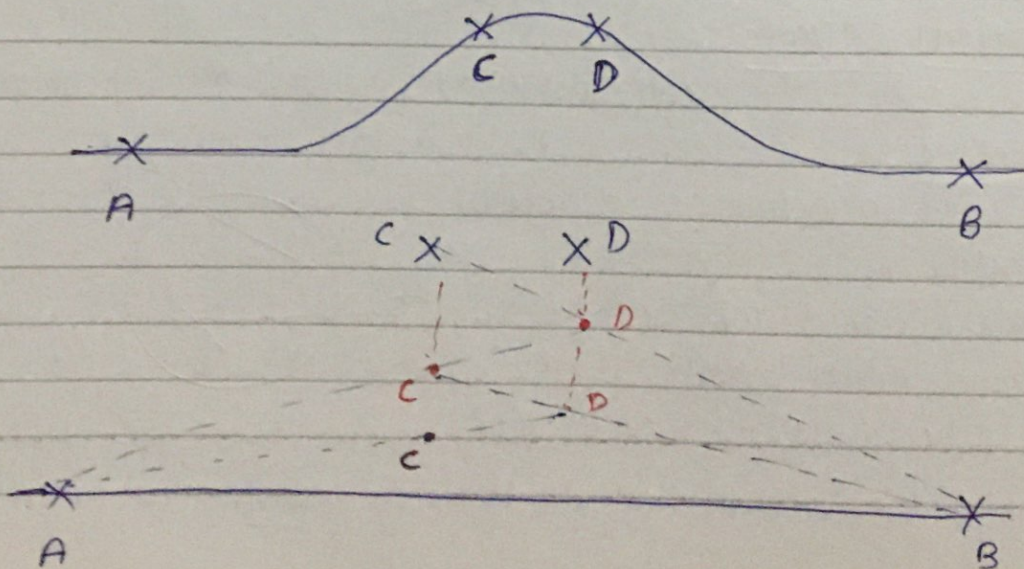
② Indirect Ranging / Reciprocal Ranging / Repeated alignment method:-

This method is used when end stations are not intervisible due to rising ground b/w them or due to long distance b/w end stations

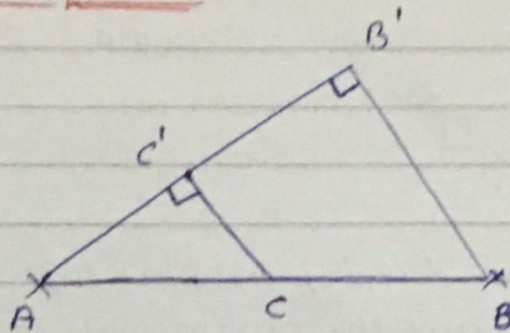
During this process we select two intermediate stations (C & D) such that

from station C $\rightarrow$ B & D are visible
from station D $\rightarrow$ A & C are visible

Mandatory condition



③ Random Line Method :-



$$\frac{AB'}{BB'} = \frac{AC'}{CC'} (?)$$

- AB is the original survey line to be established
- B' is the point from where station B is visible and line AB' is clear
- By determining AB', BB' & AC' we can determine CC' with the help of similar triangle.

Linear Measurement with chain :-

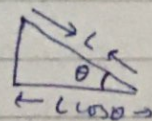
A) on smooth level surface :- If the distance to be measured is longer than chain length then by establishing intermediate points on the straight line we can determine the distance.

B) on slopping ground :-

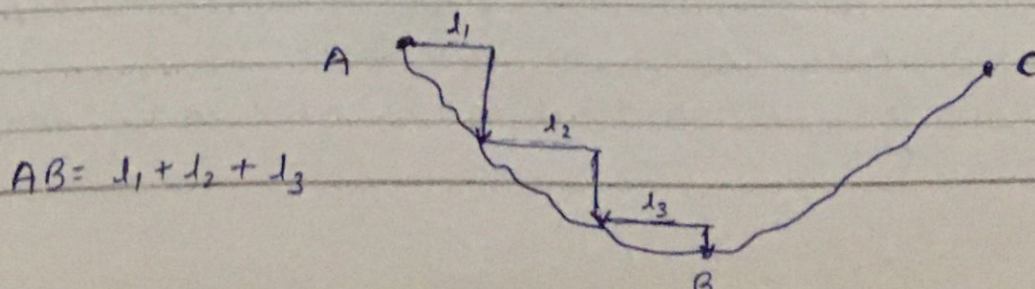
on slopping ground the horizontal distance can be measured by

① stepping

② Measuring along the slope



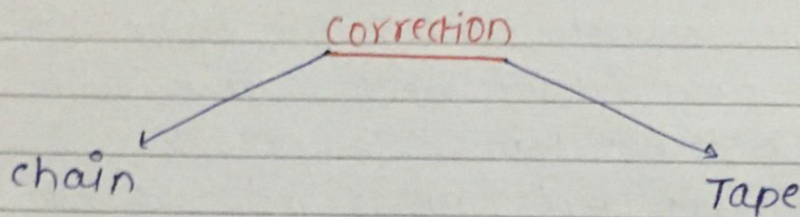
if ground slope is regular or uniform the measuring along the slope is possible but if variable slope is observed then stepping method is used.



- While stepping we follow stepping down the hill

Errors in chaining:-

- 1) Systematic error $\rightarrow$ directly proportional to length of line
- 2) Random error $\rightarrow$ directly proportional to $\sqrt{\text{length of line}}$



- ① correction for standardization
- ② correction for slop

- 1) correction for standardization
- 2) correction for slop
- 3) correction for pull
- 4) correction for temprature
- 5) correction for ~~sag~~ ~~stap~~ sag
- 6) correction for MSL
- 7) correction for mis-alignment

① correction for standardization:- $(C_a) \Rightarrow$

it is also called as correction for absolute length

- Nominal length / designated length $\rightarrow L'$ (20 m or 30 m)
- Actual length / Absolute length $\rightarrow L$
- Measured length $\rightarrow L'$
- Correct length $\rightarrow L$

- if Actual length of the tape doesn't matches with nominal length of the tape then correction for standardization is need to be done

Nominal length $\rightarrow$ which we tell !

- if Tape is actually shorter than nominal length of the tape, then measured length is greater than correct length
- Therefore error is positive and correction is Negative
- if Tape is actually Shorter than correction will be Subtractive
- Error per chain length:-

$$\text{Error per chain length} = l' - l$$

$$\text{No. of chain length} = \frac{L'}{l'}$$

$$\text{Error in M.L } (L') = (l' - l) \times \frac{L'}{l'}$$

$$\text{corr}^n \text{ in M.L } (L') \text{ i.e } C_d = (l - l') \times \frac{L'}{l'}$$

$$\text{correct length } L = L' + (l - l') \times \frac{L'}{l'}$$

$$L = L' \times l / l'$$

$$C.L = A.L \times \frac{M.L}{N.L}$$

$$C.A = \left(\frac{A.L}{N.L} \right)^2 \times M.A$$

$$C.V = \left(\frac{A.L}{N.L} \right)^3 \times M.V$$

Que. A line of true length 500 m when measured by a 20 m tape is reported to be 502 m, then correct length of the tape is

$$C.L = A.L \times \frac{M.L}{N.L}$$

$$\cancel{500} \quad 500 = \frac{502}{20} \times A.L$$

$$A.L = 19.92 \text{ m}$$

Que. Area of the plan of an old survey was plotted to a scale of 1 cm = 10 m, Now measures as 90.5 cm² as determined by planimeter. plan is found to have shrunk such that a line originally 10 cm long now measures as 9.5 cm only. A note on the plan states that the chain used was 9 cm too short. Determine true area of the survey if nominal length of the chain was 20 m.

$$\text{Area before shrinkage} = \frac{90.5}{(0.95)^2} = 100.277 \text{ cm}^2$$

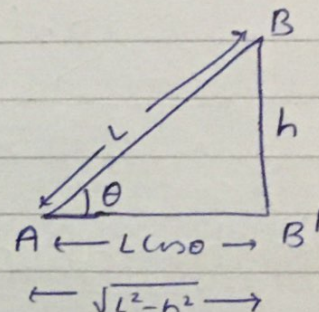
$$\text{Ground Area} = 100.277 \times 10^2 \text{ m}^2 \\ = 10027.7 \text{ m}^2$$

$$\text{Correct Area} = \left(\frac{19.91}{20} \right)^2 \times 10027.7 \\ = 9937.65 \text{ m}^2$$

② correction for slope :- $C_g \Rightarrow$

We know that slope distance is always greater than horizontal distance. Hence, correction for slope is always negative.

CASE-I Angle or slope θ is measured



$$C_g = L \cos \theta - L$$

$$\boxed{C_g = -L(1 - \cos \theta)}$$

CASE-II Height difference h is measured

$$C_g = \sqrt{L^2 - h^2} - L$$

$$C_g = \left[L \left(\sqrt{1 - \frac{h^2}{L^2}} \right) - L \right]$$

$$C_g = L \left[\left(1 - \frac{h^2}{2L^2} - \frac{h^4}{8L^4} + \dots \right) - 1 \right]$$

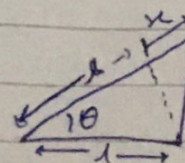
$$C_g = -\frac{h^2}{2L} - \frac{h^4}{8L^3}$$

Neglecting higher order terms

$$\boxed{C_g = -\frac{h^2}{2L}}$$

NOTE:- • if higher order terms are not neglected then magnitude of correction for slope will increase by $\frac{h^4}{8L^3}$.

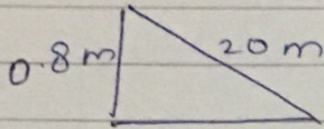
*** Hypotenious allowance



$$x = L \sec \theta - L$$

$$x = L(\sec \theta - 1)$$

Que. if downhill end of a 20 m tape is held 80 cm too low then correction for slope will be



$$C_g = \frac{-h^2}{2L} = \frac{-0.8^2}{2 \times 20} = -0.016 \text{ m}$$

③ Correction for pull; $C_p \Rightarrow$

- $P \rightarrow$ pull applied in the field
- $P_0 \rightarrow$ standard pull (60-80 N)
- $A \rightarrow$ cross-sectional Area of tape
- $E \rightarrow$ young's modulus of Elasticity
- $L \rightarrow$ Nominal length of tape or measured length of line.

$$C_p = \frac{(P - P_0)L}{AE}$$

Que. A steel tape 30 m long was standardized with a pull of 65 N. if pull at the time of measurement was 80 N. Determine correction for tape length. its weight of tape is 10 N
 $E = 2 \times 10^5 \text{ N/mm}^2$, $\gamma_{\text{steel}} = 77.10 \text{ kN/m}^3$

~~A =~~ $W = \gamma A L$

$$10 = \frac{77.1 \times 10^3}{10^9} \times A \times 30 \times 10^3$$

$$A = 4.32 \text{ mm}^2$$

$$C_p = \frac{(P - P_0)L}{AE} = \frac{(80 - 65) 30}{4.32 \times 2 \times 10^5} = +0.52 \times 10^{-3} \text{ mm}$$

④ Correction for Temperature :- $C_t \Rightarrow$

$$C_t = \alpha (t - t_0) l$$

$\alpha \rightarrow$ thermal coefficient of linear expansion

$t \rightarrow$ Field temp.

$t_0 \rightarrow$ standard temp (27°)

$l \rightarrow$ Nominal length of tape or measured length of line

- if $t > t_0$ then C_t is +ve
- if otherwise -ve

Que. if measured length of the line was 1000 m by a 30 m tape when field temperature was 30°C and standard temperature was 25°C . Determine correct length of the line if $\alpha = 10^{-5} \text{ per } ^\circ\text{C}$

Solⁿ. $C_t = \alpha (t - t_0) l$

$$\alpha = 10^{-5}, \quad l = 1000 \text{ m.}$$

$$t = 30^\circ\text{C}, \quad t_0 = 25^\circ$$

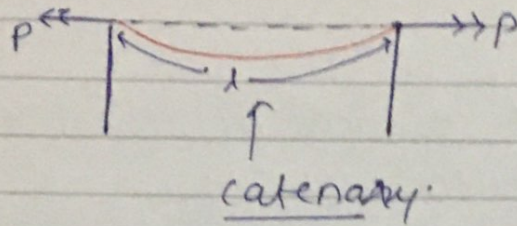
$$C_t = 10^{-5} (30 - 25) \times 10^3$$

$$= 5 \times 10^{-2} \text{ m} = +0.05 \text{ m}$$

$$C.L = 1000 + 0.05$$

$$C.L = 1000.05 \text{ m.}$$

⑤ Correction for Sag:-



- When the tape is supported b/w two ends, it takes the shape of a catenary
- if tape is standardise on flat and used in catenary correction will be **Negative** because horizontal cord length is always shorter than curved length
- For Determination of correction for sag, the curve is assumed to be parabola instead of catenary.

$$C_s = -\frac{w^2 l^3}{24 p^2} = \frac{-(wl)^2 l}{24 p^2} = -\frac{W^2 l}{24 p^2}$$

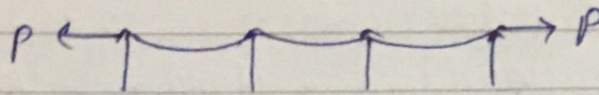
$w \rightarrow$ wt. per unit length

$l \rightarrow$ ~~ten~~ Nominal length (suspended length)

$p \rightarrow$ force applied

$W \rightarrow$ total weight of tape

- if l is total length of tape which is suspended in n number of bays
- Therefore 1 bay, $l_1 = l/n$



$$l_1 = l/n$$

$$C_s = \frac{-n \times w^2 l_1^3}{24 P^2}$$

$$C_s = \frac{-n w^2 (l/n)^3}{24 P^2} = \frac{-w^2 l^3}{24 n^2 P^2}$$

$$C_s = \frac{-W^2 \times l}{24 n^2 P^2}$$

NOTE:- If tape is standardise in catenary and used for flat then correction for sag will be positive.

→ Normal tension P_n is a theoretical pull at which correction for pull is numerically equal to correction for sag.

$$\frac{W^2 l}{24 P_n^2} = \frac{(P_n - P_0) l}{AE}$$

Que. Determine sag correction for 30 m steel tape under a pull of 80 N in 3 bays of 10 m each. c/s Area of tape is 8 mm² and unit wt of steel may be taken 77 kN/m³

$$W = \gamma A L = \frac{77 \times 10^3 \times 8 \times 30 \times 10^3}{10^9} = 18.48 \text{ N}$$

$$C_s = \frac{-18.48^2 \times 30}{24 \times 3^2 \times 80^2} = -7.41 \times 10^{-3} \text{ m}$$

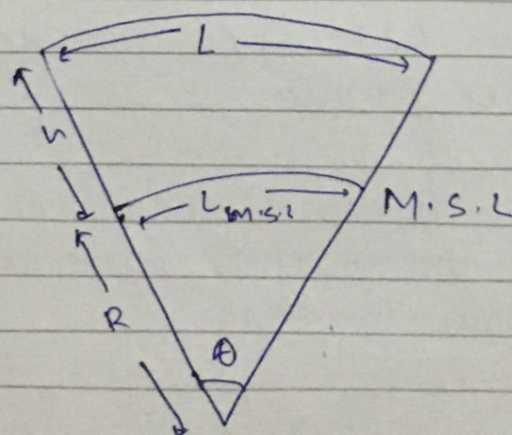
Que. Determine normal tension for a steel tape supported b/w two supports 10 m apart if standard tension is 65 N and weight of tape per meter is 0.62 N. $E = 2 \times 10^5 \text{ N/mm}^2$ and c/s Area of tape is 8 mm^2

$$\frac{W^2 l}{24 P_n^2} = \frac{(P_n - P_0) l}{AE}$$

$$\frac{(0.62 \times 10)^2}{24 \times P_n^2} = \frac{(P_n - 65)}{8 \times 2 \times 10^5}$$

$$P_n = 162.29 \text{ N}$$

⑥ Correction for MSL:-



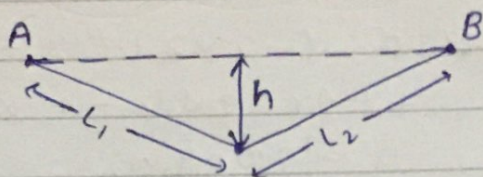
$$\begin{aligned} C_{MSL} &= L_{MSL} - L \\ &= R\theta - (R+h)\theta \\ &= -h\theta \end{aligned}$$

$$C_{MSL} = \frac{-hL}{R+h}$$

$$h \ll R$$

$$C_{MSL} = \frac{-hL}{R}$$

⑦ Correction for Mis-Alignment :-



$$C_m = (\sqrt{L_1^2 - h^2} + \sqrt{L_2^2 - h^2}) - (L_1 + L_2)$$

Que: During the Measurement in CATENARY of a Survey line of 4 bays following info was obtained.

Bays	M.L(m)	t°C	T(N)	Diff. in elevation b/w two ends of tape Δh
1	29.899	18	178	+0.064
2	29.901	18	178	+0.374
3	29.882	18.1	178	-0.232
4	29.950	17.9	178	+0.238

Mass of tape $M \rightarrow 0.026 \text{ kg/m}$, C/s Area $= 3.24 \text{ mm}^2$

$t_o \rightarrow 20^\circ\text{C}$, $P_o \rightarrow 89 \text{ N}$, $\alpha = 1.1 \times 10^{-5} / ^\circ\text{C}$

$E \rightarrow 2.07 \times 10^5 \text{ MN/m}^2$, Mean level of the line is 26.89 m above M.S.L

Determine Absolute Length of Line Reduce to M.S.L if Radius of earth is 6370 km

Solⁿ

$$C_p = \frac{(P - P_o)}{AE} \times L = \frac{(178 - 89)}{3.24 \times 2.07 \times 10^5} \times 119.632$$

$$= +0.016 \text{ m}$$

$$\begin{aligned}
 C_t &= \alpha \sum (t - t_0) l \\
 &= 1.1 \times 10^{-5} (-2 \times 29.899 - 2 \times 29.901 \\
 &\quad - 1.9 \times 29.882 - 2.1 \times 29.950) \\
 &= -0.003 \text{ m}
 \end{aligned}$$

$$\begin{aligned}
 C_s &= \cancel{\frac{W^2 l}{24 n^2 p^2}} \quad \frac{-w^2 l}{24 p^2} \times \sum l^3 \\
 &= - \frac{(0.026 \times 9.81)^2}{24 \times 178^2} \times [29.899^3 + 29.901^3 \\
 &\quad + 29.882^3 + 29.950^3] \\
 &= -0.009 \text{ m}
 \end{aligned}$$

$$C_g = - \sum \frac{(\Delta h)^2}{2L} = -0.004 \text{ m}$$

$$C_{msl} = - \frac{hL}{R} = \frac{-26.89 \times 119.632}{6370 \times 10^3} = -0.001 \text{ m}$$

$$\begin{aligned}
 C.L &= 119.632 + 0.016 - 0.003 - 0.009 \\
 &\quad - 0.004 - 0.001
 \end{aligned}$$

$$C.L = 119.631 \text{ m}$$

Field Work in chain Surveying:-

① Reconnassiance Survey:- It is preliminary inspection of the Area to be surveyed to get some idea of the terrain and to identify principle features of the ground

• It helps the surveyer to decide upon best possible arrangement of triangles.

② Marking of stations:- After completion of Reconnaissance survey best suitable position of main survey station are marked on the ground such that they can be easily identified afterward if required.

③ Running of survey lines:- Survey lines are run to measure distance b/w main station and to locate adjacent detail by taking offsets.

④ Taking offsets:-

NOTE:- Max. length of offset depends on accuracy desire, Nature of ground, scale of plotting.

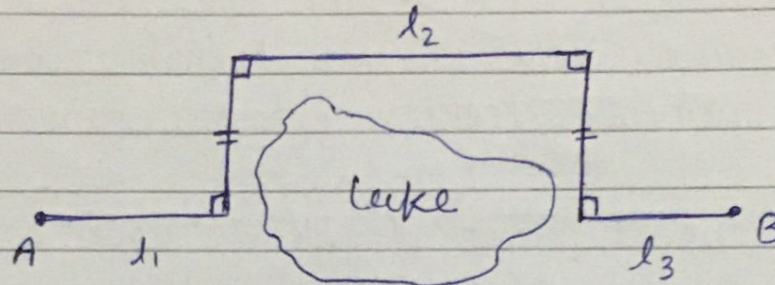
• Degree of accuracy of offset depends on scale of plotting, length of offset, importance of detail whose information need to be collected.

① Reconnaissance → preliminary → Detail survey

Obstacle in chaining :-

Case-I chaining round of obstacle is possible

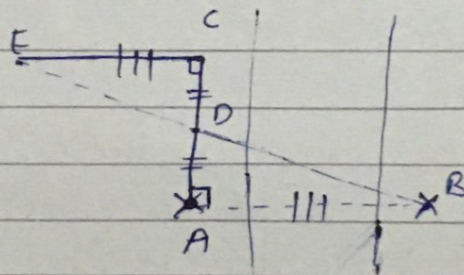
Ex: lake, pond etc



$$AB = l_1 + l_2 + l_3$$

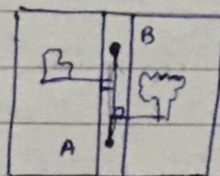
Case-II chaining round of obstacle is not possible

Ex: River

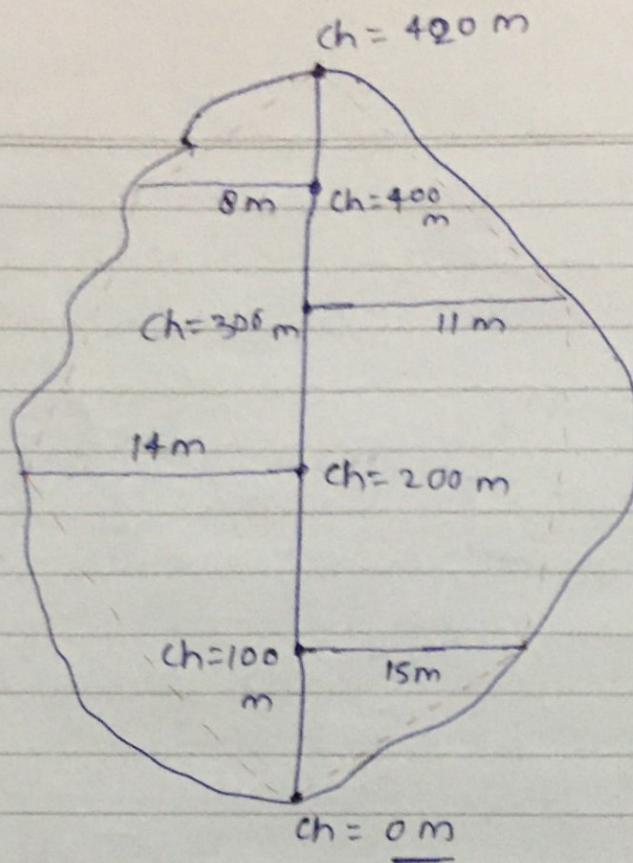


Case-III Recording of Measurement

Double line field book is used to record chain survey and recording starts from bottom toward top



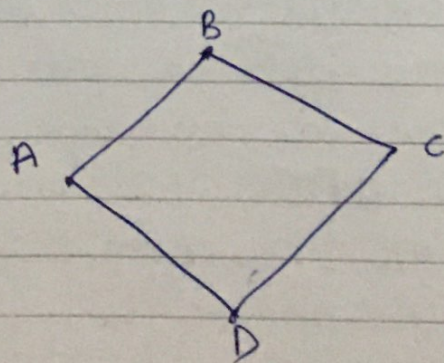
Cross staff survey :- It is a special type of chain surveying which is used to locate boundary of the field for calculating approx. Area of field



- During this survey Area is plotted on a suitable scale

③ COMPASS SURVEYING

- Compass Surveying is the branch of Surveying in which dirⁿ of survey lines are established with the help of compass and length is measured with the help of chain or Tape.
- Compass are generally used to run a traverse in the field.
- In surveying traverse is a frame work consisting of series of straight lines forming a open or closed polygon
- points such as A & B are called as Traverse station or Traverse point.
- At each traverse station where sides meet Angles are measured
- Depending on instrument used for the measurement of angles traverse can be clasified as Compass traverse, plane table traverse, theodolite traverse.



Difference b/w chain surveying & Traverse surveying :-

Chain surveying

- ① Only linear measurements are done.
- ② Framework consist of network of triangle.
- ③ Check lines or proof lines are required
- ④ used for smaller Area and Accuracy desired is low

Traverse surveying

Linear & Angular measurements are done.

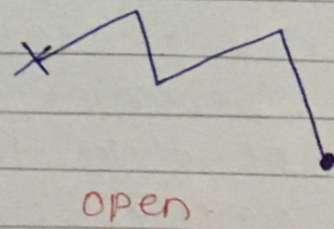
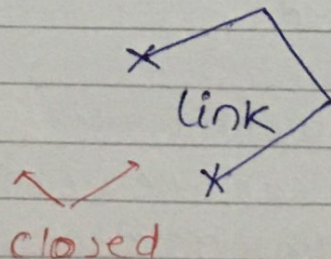
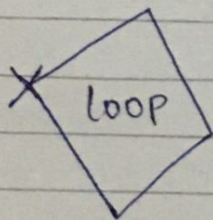
Framework consist of open or closed polygon.

Not Required

As Accuracy is checked by Method of adjustment.

Used for Larger Area Accuracy desired is high.

TYPES OF TRAVERSE :-



- ① CLOSED Traverse :- It is a traverse which starts from a point of known location and closes either at the same point or at another point of known location.

- ② Open Traverse :- It starts from a point of known location and closes at another point of unknown location.

- open traverse is used for linear projects.

- An open traverse can be checked by method of Sods or by Astronomical observations

MEASUREMENT OF ANGLES :- Direction of survey line can be defined in two ways

- 1) Relative to each other
- 2) Relative to some fixed reference direction

- This fixed reference direction in the language of surveying is called as meridian.
- There are 4 types of Meridians.

1) TRUE MERIDIAN :-

True Meridian at a point is the line joining true north and true south of the earth surface through that point.

- True Meridian is determined with the help of astronomical observation of SUN & STAR.

- True Meridian at a place does not change with the passage of time.

2) MAGNETIC MERIDIAN :-

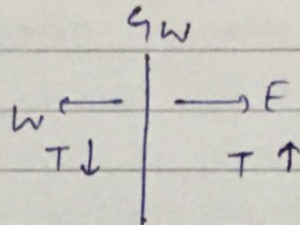
- Magnetic Meridian at a point is the direction indicated by a freely suspended magnetic bar or needle, provided that it is not affected by local magnetic field.

- Magnetic meridian at a place is not fix, it changes with the passage of time.

③ Grid Meridian:-

$$\frac{360^\circ}{24} \Rightarrow 1 \text{ hr} \rightarrow 15^\circ$$
$$30 \text{ Min} \rightarrow 7.5^\circ$$

$$\frac{82.5^\circ \text{ E}}{7.5} = 11$$



For survey of a country the true meridian passing through central place taken as reference meridian for the whole country and such a reference meridian is called as Grid Meridian

Ex: 82.5° E Allahbad.

④ Arbitrary Meridian:-

It is the meridian taken in arbitrary direction.

- Generally it is taken in the direction from a traverse station to a well defined point such as top of chimney, Tower etc.

- Sometimes it is also direction of first traverse line.

BEARING:-

- Bearing is the horizontal angle b/w a survey line and reference meridian

- Bearings can be True Bearing, Magnetic Bearing, Grid Bearing or arbitrary bearing

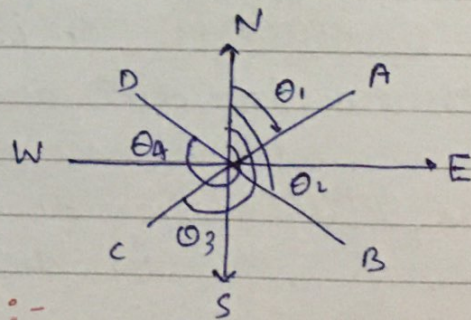
- True bearing of a survey line does not change with the passage of time whereas magnetic bearing changes with the passage of time because magnetic meridian changes

NOTE For all important surveys True bearing is preferred over Magnetic Bearing

Designation of Bearing :-

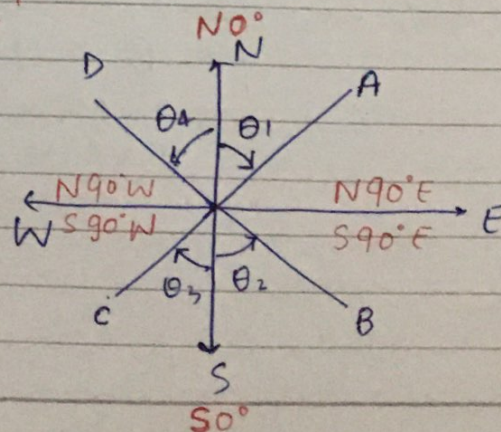
① Whole Circle Bearing System :- WCB

- WCB is horizontal clockwise angle measured from North end of the reference meridian towards the survey line
- It is also called an AZIMUTH
- It ranges b/w $0-360^\circ$
- prismatic compass



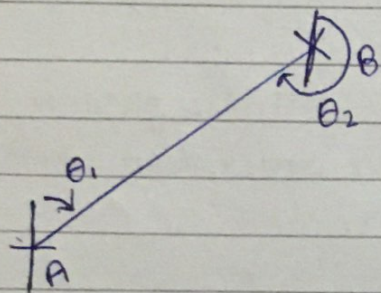
② Quadrantal Bearing system :-

Line	Q.B	(Reduce Bearing)
OA	$N\theta_1 E$	
OB	$S\theta_2 E$	
OC	$S\theta_3 W$	
OD	$N\theta_4 W$	



- It is the acute angle b/w survey line and reference meridian.
- It can be measured from North or south end in clockwise or Anti-clockwise direction.
- Its value ranges b/w $0 - 90^\circ$.
- Surveyer compass.

FORE BEARING & BACK BEARING :-



Line	F.B	B.B
AB	θ_1	θ_2
BA	θ_2	θ_1

Fore Bearing :- It is the bearing of line measured in dirⁿ of progress of survey.

BACK BEARING :- It is bearing of a line measured in dirⁿ opposite to the progress of survey.

$$B.B = F.B \pm 180^\circ$$

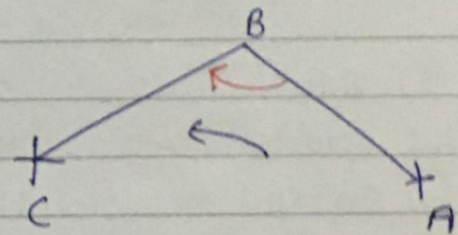
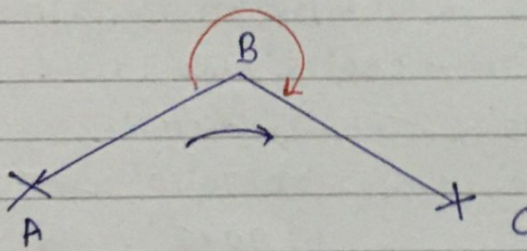
+ve $\rightarrow F.B < 180^\circ$
-ve $\rightarrow F.B > 180^\circ$

gf Fore Bearing of a line is given in Quadrantal Bearing system then Back bearing can be obtained by replacing North by South, South by North, East by West & West by East also keeping same value of θ

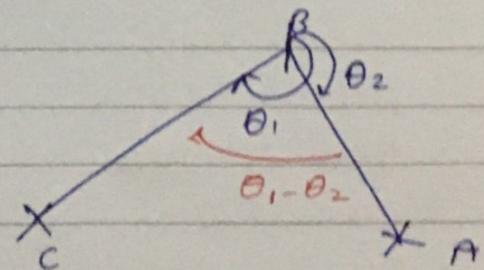
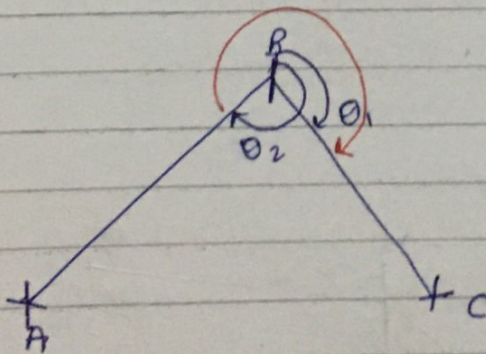
Ex. $S 50^\circ E \rightarrow N 50^\circ W$
 $N 40^\circ W \rightarrow S 40^\circ E$

Included Angles :-

Included angle is angle measured in clockwise dirⁿ from previous line to the next line



CALCULATION OF INCLUDED Angle from BEARING:-

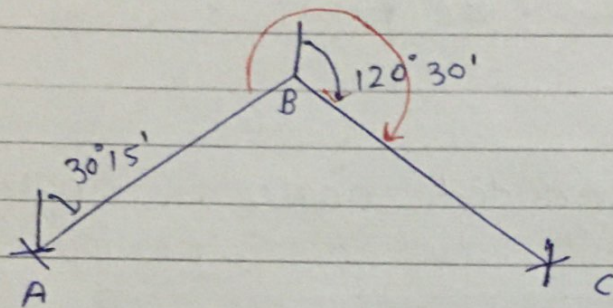


Intended angle = F.B of Next line - B.B of previous line

Intended angle = F.B of Next line - B.B of previous line
 $+ 360$

If a Negative Value is obtained then simply add 360° to get required value of included angle.

Que. WBC of line AB & BC are $30^\circ 15'$ & $120^\circ 30'$ respectively. What is included angle, $\angle B$ or angle ABC?



$$\angle B = 59^\circ 45' + 90^\circ + 120^\circ 30' = 270^\circ 15'$$

$$\angle B = 120^\circ 30' - (30^\circ 15' + 180^\circ) + 360 = 270^\circ 15'$$

- | | | |
|----------------------|---------------------|----------------------|
| a) $89^\circ 45'$ | a) $89^\circ 45'$ ✓ | a) $89^\circ 40'$ |
| b) $270^\circ 15'$ ✓ | b) $270^\circ 10'$ | b) $270^\circ 10'$ ✓ |
| c) $90^\circ 15'$ | c) $90^\circ 15'$ | c) $90^\circ 15'$ |
| d) $269^\circ 45'$ | d) $269^\circ 45'$ | d) $269^\circ 45'$ |

Que. determine the value of included angle in a close compass traverse

Line F.B

AB 40°

BC 70°

CD 210°

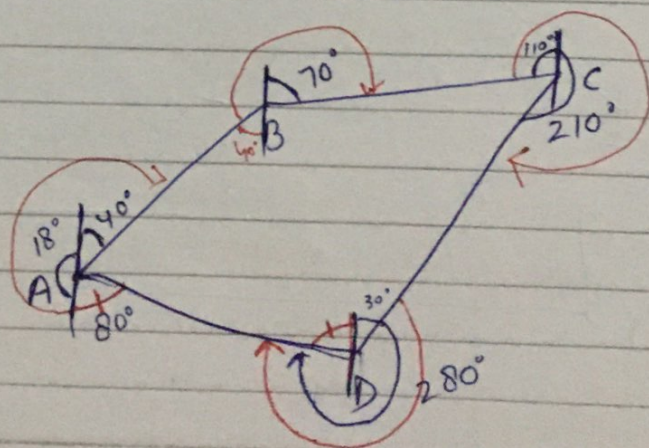
DA 280°

$$\angle A = 40 + 180 + 80 = 300^\circ$$

$$\angle B = 180 - 40 + 70 = 210^\circ$$

$$\angle C = 110^\circ + 210^\circ = 320^\circ$$

$$\angle D = 280^\circ - 30^\circ = 250^\circ$$



By formula.

$$\angle A = 40^\circ - (280^\circ - 180^\circ) + 360^\circ = 300^\circ$$

$$\angle B = 70^\circ - (40^\circ + 180^\circ) + 360^\circ = 210^\circ$$

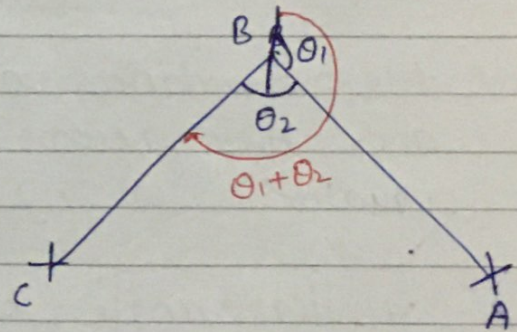
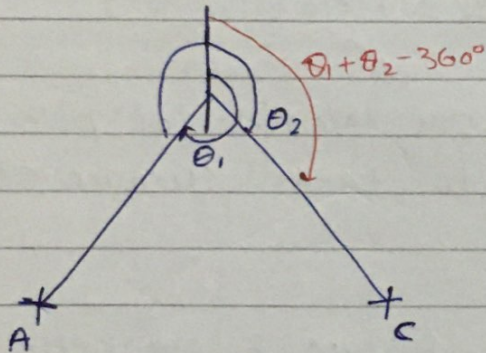
$$\angle C = 210^\circ - (70^\circ + 180^\circ) + 360^\circ = 320^\circ$$

$$\angle D = 280^\circ - (210^\circ - 180^\circ) = 250^\circ$$

Sum of exterior angles $\rightarrow (2n+4) 90^\circ$

Sum of interior angles $\rightarrow (2n-4) 90^\circ$

CALCULATION OF BEARING FROM INCLUDED ANGLE:-



F.B OF Next line = B.B OF previous angle + included angle

if The value obtained is more than 360° then subtract 360° to get the required value

Que. Following included angles were observed in an open traverse angle $\angle ABC = 89^\circ 45'$, $\angle BCD = 125^\circ 22'$
 $\angle CDE = 135^\circ 34'$

Calculate Bearings of all the lines if

F.B of AB $\rightarrow 150^\circ 30'$

Solⁿ

$$\text{F.B of BC} \rightarrow (150^\circ 30' + 180^\circ) + 89^\circ 45' - 360^\circ = 60^\circ 15'$$

$$\text{F.B of CD} \rightarrow (60^\circ 15' + 180^\circ) + 125^\circ 22' - 360^\circ = 5^\circ 37'$$

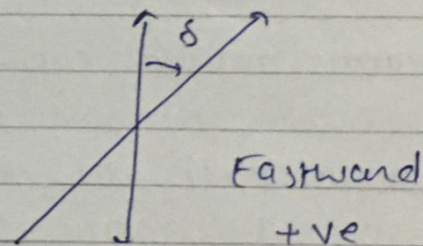
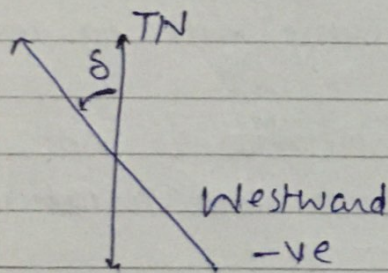
$$\text{F.B of DE} \rightarrow (5^\circ 37' + 180^\circ) + 135^\circ 34' = 321^\circ 11'$$

MAGNETIC FIELD OF EARTH :-

- Earth act as a powerful magnet with its magnetic lines of forces running from magnetic South towards magnetic North.
- A magnetic Needle aligns itself in the dirn parallel to earth magnetic field.
- Magnetic lines of forces are verticle at poles and they become parallel to earth surface at equator.
- Vertical angle b/w earth surface & magnetic lines of forces is called as **Angle of Dip**
- At poles $\text{Dip} = 90^\circ$
At Equator $\text{Dip} = 0^\circ$
B/w Equator & poles $\text{Dip} = 0-90^\circ$
- Lines joining points of equal angle of Dip on the earth surface is called as **Isoclinic line**
Ex: Latitude
- A special case of isoclinic line when angle of Dip is zero is called as **Aclinic line**
Ex: Equator

MAGNETIC DECLINATION :-

- Generally Magnetic Meridian and True Meridian don't coincide with each other
- Horizontal angle b/w True Meridian & Magnetic Meridian at the time of observation is called as **Magnetic Declination**
- Magnetic Declination in the field can be determined by observing Magnetic Bearing and True Bearing of a Survey line.
- Declination at a place changes from time to time
- Declination at a time changes from place to place



- Variation of Declination is shown by Isogonic line
- Isogonic Line $\Rightarrow$

These are the lines joining points of same declination on the earth surface at a given point of time.

- AGONIC Line $\Rightarrow$ These are special Isogonic line which pass through points having zero declination. at all the points on Agonic line magnetic meridian coincides with true meridian.

VARIATION OF MAGNETIC DECLINATION :-

There are

4 types of variation which occur in Magnetic Declination of a place.

1) Secular Variation :-

- Secular variation of declination occurs continuously over a long period of time.
- This variation approximately follows sin curve over a period of 300 years.
- There is no reliable method of predicting secular variation.

2) Angular Variation :-

- Angular variation is variation of declination in a year.
- It is caused due to revolution of earth around sun.
- Angular variation is not same as annual rate of change of secular variation.

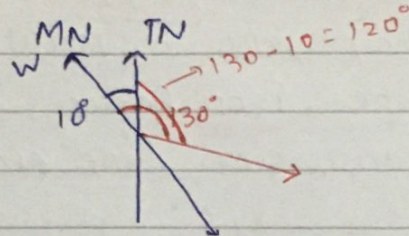
③ Diurnal Variation :-

- Diurnal variation is variation of declination in one day.
- It is caused due to ~~revolution~~ rotation of earth about its own axis.
- It is observed that diurnal variation is
 - 1) Greater in day than in night
 - 2) Greater in summer than winter
 - 3) Greater at poles than at equator
 - 4) It changes from year to year

④ Irregular Variation :- It is caused due to volcanic eruption, earthquake or storm.

Que: Magnetic Bearing of a survey line AB is 130°
determine True bearing of the same line if magnetic declination is $10^\circ W$

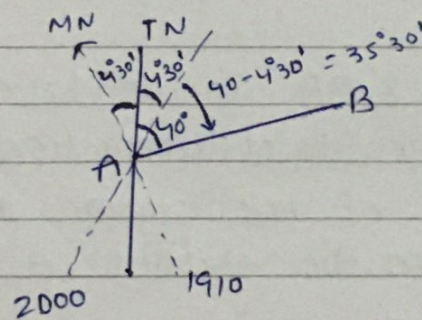
Solⁿ



Que: if a line AB had Magnetic bearing of $44^\circ 30'$
in year 1910 When declination was $4^\circ 30'$ West.
determine magnetic bearing of same line in year 2000
When Annual rate of change of declination is $6' E$

Solⁿ

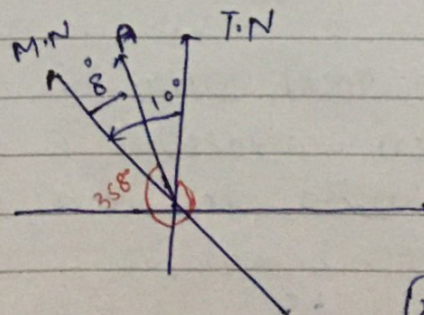
1910 $\rightarrow$ 2000 $6' E / \text{year} \rightarrow$ ^{change in} total declination = $9^\circ E$



Magnetic Bearing = $35^\circ 30'$

Que: if True bearing of a survey station A = 358°
and magnetic bearing of the same was 8°
Fore Bearing of AB When measured by prismatic
Compass = 300° Determine True fore bearing of AB?

Solⁿ



True Bearing = $300^\circ - 10^\circ$
 $= 290^\circ$
290

LOCAL ATTRACTION:

local attraction is attraction of magnetic needle to a local magnetic field other than earth magnetic field.

It is caused by any iron material nearby compass station ex: Iron pipe, Iron fence, steel vehicle, iron deposit, transmission line etc.

Under the influence of local magnetic field, magnetic needle will deviate and magnetic bearing of the survey line may change.

- To determine local attraction it is mandatory to take fore bearing and back bearing of each survey line.
- The difference b/w FB & BB should be equal to 180°
- If $FB - BB \neq 180^\circ$ then local attraction is expected at
 - 1) either of the stations A or B
 - 2) at both the stations A & B
- If $FB - BB = 180^\circ$ then both the stations are free from local attraction.

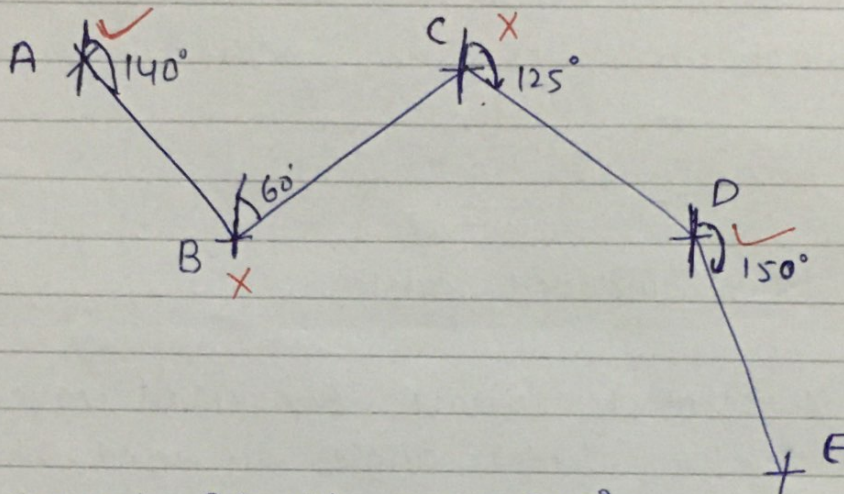
CORRECTION FOR LOCAL ATTRACTION:-

Method 1

• By calculating local attraction @ each station in this method local attraction @ each station is determined and then required correction is applied to observed bearings.

Example.

Line	FB	BB
AB	140°	318°
BC	60°	240°
CD	125°	302°
DE	150°	



$\therefore$ Correct BB of AB = 318°

$\therefore$ Correct FB of AB = $318^\circ - 180^\circ = 138^\circ$

Obs. FB of AB = 140°

therefor correct @ stⁿ A = -2°

$\therefore$ Correct FB of CD = 125°

$\therefore$ correct BB of CD = $125 + 180^\circ = 305^\circ$

$\therefore$ observed BB of CD = 302°

$\therefore$ correction @ st D = $+3^\circ$

since observed FB of DE = 150°

$\therefore$ correct FB of DE = $150 + 3^\circ$
 $= 153^\circ$

STEPS

- ① Observe a line whose FB & BB Differ by 180°
- ② End stations of such line are free from local attraction
- ③ With the help of correct bearing and correction for local attraction we can correct the bearing at all the other stations.

Observation

Correction for all the bearings are taken constant at each and every station

- This Method is suitable for open traverse
- This Method fails in case of close traverse that is When instrumental errors are present or theoretical sum does not matches with Actual sum.

METHOD-2 • By included angles

- This Method is most suitable for closed traverse
- In this Method included angles are used to determine correct bearings
- Since all the bearings taken at a station are equally affected in the same dirⁿ Hence, included angles are free from local attraction

STEPS :-

- ① Determine included angles
- ② observe a line whose FB & BB differ exactly by 180°
- ③ With the help of correct bearing and included angles we can determine correct bearing at all the other stations.

Que : If observed bearing are as given below then determine included angles.

Line	FB	BB
AB	$150^{\circ}30'$	$329^{\circ}45'$
BC	$78^{\circ}0'$	$256^{\circ}30'$
CD	$42^{\circ}30'$	$223^{\circ}45'$
DE	$315^{\circ}45'$	$134^{\circ}15'$
EA	$220^{\circ}15'$	$40^{\circ}15'$

$$\angle A = 150^{\circ}30' - 40^{\circ}15' = 110^{\circ}15'$$

$$\angle B = 78^{\circ}0' - 329^{\circ}45' + 360^{\circ} = 108^{\circ}15'$$

$$\angle C = 42^{\circ}30' - 256^{\circ}30' + 360^{\circ} = 146^{\circ}0'$$

$$\angle D = 315^{\circ}45' - 223^{\circ}45' = 92^{\circ}0'$$

$$\angle E = 220^{\circ}15' - 134^{\circ}15' = 86^{\circ}0'$$

$$\begin{aligned}\text{Sum of all angles} &= 110^{\circ}15' + 108^{\circ}15' + 146^{\circ} + 92^{\circ} + 86^{\circ} \\ &= 542^{\circ}30'\end{aligned}$$

$$\text{theoretical sum} = (2n-4)90^{\circ} = 540^{\circ}$$

- We can observe that theoretical sum and actual sum don't match with each other and this discrepancy is due to instrumental error.

$$\bullet \text{ Error} = M.V - F.V$$

$$\therefore \text{Error} = +2^{\circ}30'$$

$$\therefore \text{Correction per angle} = -30'$$

$$\begin{aligned}\therefore \text{Corrected angles} \quad \angle A &= 109^{\circ}45' & \angle D &= 91^{\circ}30' \\ \angle B &= 107^{\circ}45' & \angle E &= 85^{\circ}30' \\ \angle C &= 145^{\circ}30'\end{aligned}$$

Que A compass traverse ABCD was run and following bearings were observed determine stations affected by local attraction and correct the bearings by both the methods

Line	AB	BB	FB	BB
AB	$80^{\circ}45'$	260°	$81^{\circ}5'$	$261^{\circ}5'$
BC	$130^{\circ}30'$	$311^{\circ}35'$	$131^{\circ}35'$	$311^{\circ}35'$
CD	$240^{\circ}15'$	$60^{\circ}15'$	$240^{\circ}15'$	$60^{\circ}15'$
DA	$290^{\circ}30'$	$110^{\circ}10'$	$290^{\circ}30'$	$290^{\circ}30' - 180^{\circ} = 110^{\circ}10'$

Solⁿ We can observe that FB & BB of line CD are differing by 180° so the stations C & D are free from local attraction. Also A & B are affected by local attraction.

Method 1

We can observe all the bearings taken from C & D would be correct

Correction @ station A = $+20'$

Since observed FB of AB = $80^{\circ}45'$

$\therefore$ correct FB of AB = $81^{\circ}5'$

$\therefore$ correct BB of AB = $81^{\circ}5' + 180^{\circ}$
= $261^{\circ}5'$

Observed BB of AB = 260°

so correction @ B = $261^{\circ}5' - 260^{\circ}$
= $+1^{\circ}5'$

$\therefore$ observed FB of BC = $130^{\circ}30'$

Correct FB of BC = $131^{\circ}35'$

Method 2.

$$\angle A = 330^{\circ}35'$$

$$\angle B = 230^{\circ}30'$$

$$\angle C = 288^{\circ}40'$$

$$\angle D = 230^{\circ}15'$$

$$\text{Actual sum} = \angle A + \angle B + \angle C + \angle D = 1080^{\circ} = \text{theoretical sum}$$

$$\text{Theoretical sum} = (2n+4)90^{\circ} = 1080^{\circ}$$

$$\therefore \text{corr't F.B of DA} = 290^{\circ}30'$$

$$\therefore \text{corr't BB of DA} = 110^{\circ}30'$$

$$\therefore \text{corr't F.B of AB} = 110^{\circ}30' + 330^{\circ}35' - 360^{\circ} = 81^{\circ}5'$$

$$\therefore \text{corr't BB of AB} = 261^{\circ}5'$$

$$\begin{aligned} \text{corr't FB of BC} &= 261^{\circ}5' + 230^{\circ}30' - 360^{\circ} \\ &= 131^{\circ}35' \end{aligned}$$

$$\text{corr't BB of BC} = 311^{\circ}35' \text{ (OK)}$$

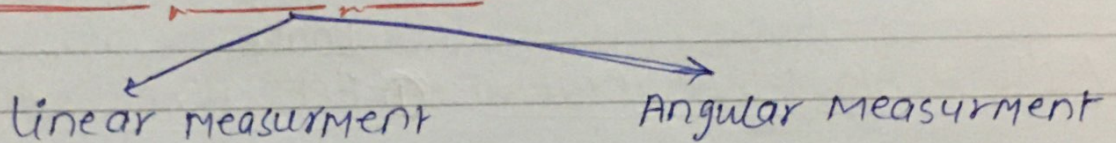
④ TRAVERSING

Traverse is a frame work consisting of straight line . forming a closed and open polygon.

Types of Traverse :-

- 1) closed traverse
- 2) open traverse

MEASUREMENTS IN TRAVERSING :-



- 1) chain or tape
- 2) theodolite
- 3) EDM

- 1) loose needle method
- 2) fast needle method
- 3) included angle method
- 4) deflection angle method
- 5) direct angle method

In traversing linear measurement equipment should be selected such that degree of accuracy is of the same order of angular measurement

If very precise equipments are used for angular measurement then equally precise equipment shall be used for distance measurement.

If θ is least count of angular measurement equipment then Linear measurement equipment should have error of the order of

$$\tan \theta = \frac{Sl}{l}$$

Que. If least count of compass is $30'$. Find order of error permissible in linear measurement equipment used in compass survey.

$$\tan 30' = \frac{\delta l}{l} = 0.00873$$

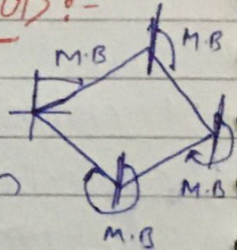
if $l = 1 \text{ m} = 1000 \text{ mm}$

therefore $\delta l = 8.73 \text{ mm}$.

LOOSE

ANGULAR MEASUREMENT ① FREE NEEDLE METHOD:-

In this method Magnetic Bearing of each traverse line is measured at every station



② FAST NEEDLE METHOD:-

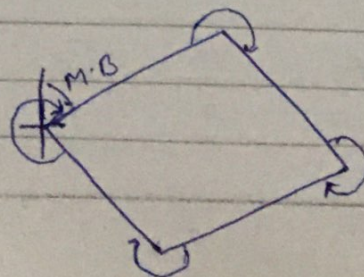
In this method Magnetic Bearing is determined for one traverse line which is generally the first traverse line and included angles are determined for other traverse line



This method is more accurate than previous method.

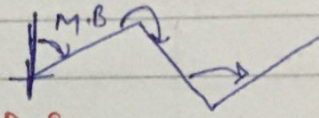
③ INCLUDED ANGLE METHOD:-

In this method Magnetic Bearing of any one line is measured which is generally first traverse line and included angles are measured at all the stations.

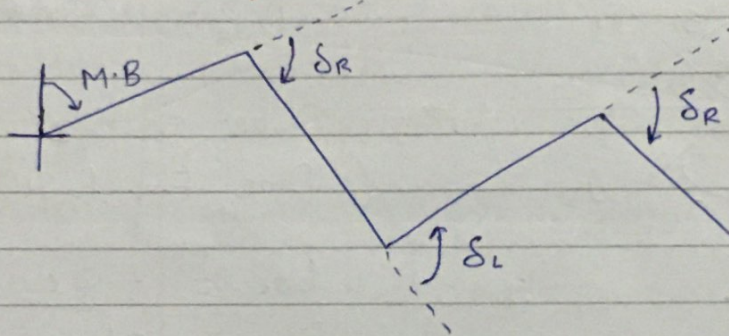


④ DIRECT ANGLE METHOD :-

This Method is generally used in open traverse.
• In this Method direct angles towards Right are measured



⑤ DEFLECTION ANGLE METHOD :-



It is used for open traverse in which the value of deflection angle is small like canal, pipeline, sewer etc.

ANGLE OF MISCLOSURE :-

In loop traverse the difference b/w Actual sum of measured angles and theoretical sum is called as Angle of Misclosure.

It's permissible limit is

$$E = K\sqrt{N}$$

Where $N \rightarrow$ number of sides / Angles

$K \rightarrow$ It is a constant whose value

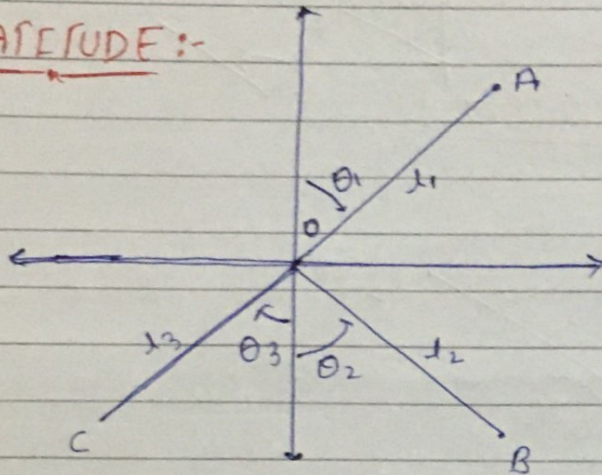
depends on least count of theodolite,

Number of repetition and accuracy desired

Generally for ordinary theodolite survey
 $K = 20''$

LATITUDE AND DEPARTURE :-

LATITUDE :-



$$L_{OA} = l_1 \cos \theta_1$$

$$D_{OA} = l_1 \sin \theta_1$$

$$L_{OB} = -l_2 \cos \theta_2$$

$$D_{OB} = l_2 \sin \theta_2$$

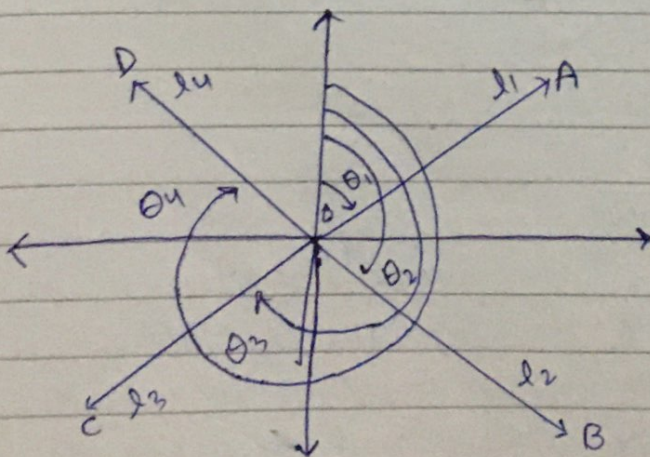
$$L_{OC} = -l_3 \cos \theta_3$$

$$D_{OC} = -l_3 \sin \theta_3$$

Orthographic projection of a line on N-S meridian
 positive $\rightarrow$ Northing, Negative $\rightarrow$ Southing

DEPARTURE :- Orthographic projection of a line on
 E-W meridian

+ve $\rightarrow$ Easting, -ve $\rightarrow$ Westing



$$L_{OA} = l_1 \cos \theta_1$$

$$D_{OA} = l_1 \sin \theta_1$$

$$L_{OB} = l_2 \cos \theta_2$$

$$D_{OB} = l_2 \sin \theta_2$$

$$L_{OC} = l_3 \cos \theta_3$$

$$D_{OC} = l_3 \sin \theta_3$$

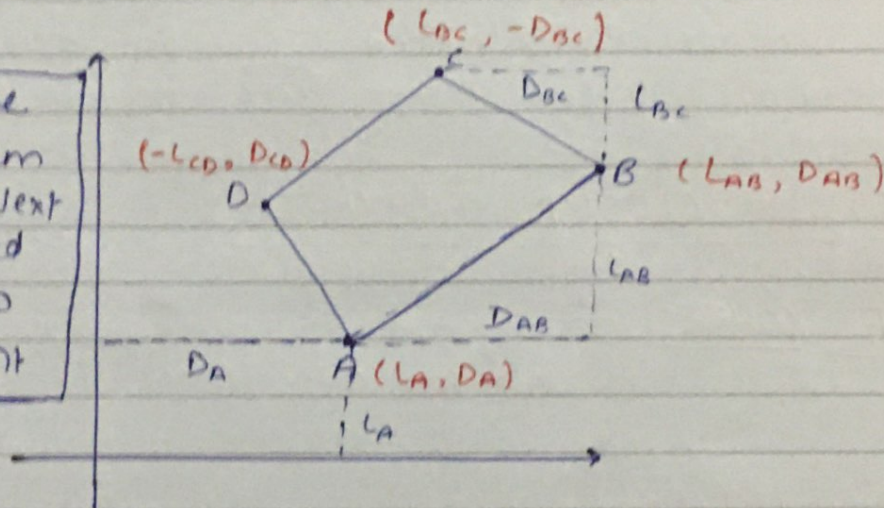
$$L_{OD} = l_4 \cos \theta_4$$

$$D_{OD} = l_4 \sin \theta_4$$

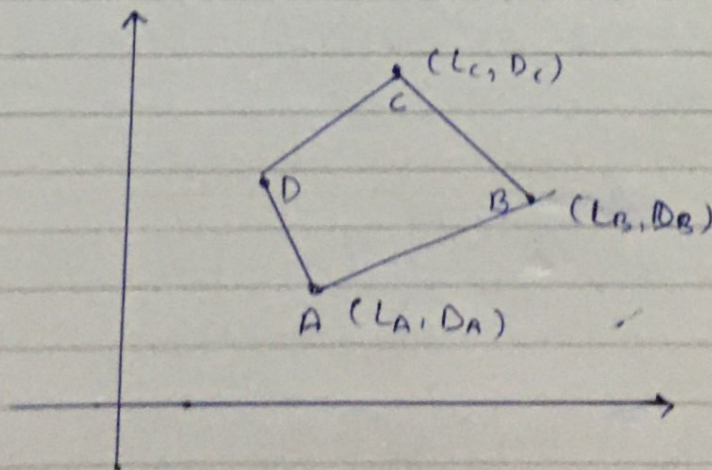
NOTE In case of WCB Algebraic sign of latitude and departure is given by $\sin\theta$ and $\cos\theta$ itself.

CONJUGATIVE / DEPENDENT COORDINATES:-

In conjugative coordinate system coordinate of Next point is defined with respect to previous point



INDEPENDENT COORDINATE:-



$$L_C = L_A + L_{AB} + L_{BC}$$

$$D_C = D_A + D_{AB} + D_{BC}$$

If coordinate of any point of the traverse is defined with respect to common origin then it is called as independent coordinate system. These are also called as total Latitude or total Departure.

- Total latitude or departure of any point
 = Latitude or departure of initial point
 + Algebraic sum of latitude and departure
 of all the line upto that point

NOTE :- In independent coordinate error is localised whereas in conjugative coordinate system error gets maximized.

CHECKS IN CLOSED TRAVERSE :-

1) LOOP TRAVERSE :-

$$\sum L = 0$$

$$\sum D = 0$$

2) LINK TRAVERSE :-

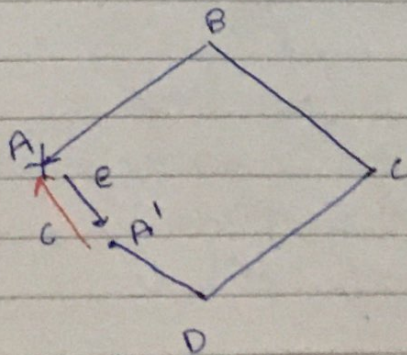
$\sum L$ = Latitude of last point - Latitude of First point

$\sum D$ = Departure of Last point - Departure of First point

CLOSING ERROR / ERROR OF CLOSER / CLOSING LINE :-

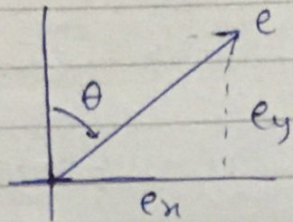
If a traverse has error of closer, it will not close when plotted on paper

CASE - I LOOP Traverse :-



closing error = AA'

that is coordinate of A' with respect to starting point



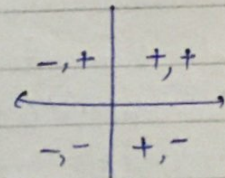
$$e_x = \sum D$$

$$e_y = \sum L$$

$$e = \sqrt{e_x^2 + e_y^2}$$

$$\tan \theta = e_y / e_x$$

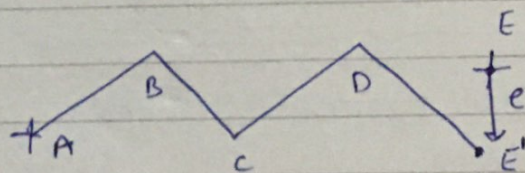
$$\theta = \tan^{-1} \frac{|e_x|}{|e_y|}$$



The quadrant of closing error will be decided with the help of algebraic sign of e_x & e_y

Magnitude of correction is same as magnitude of closing error but direction is opposite.

LINK TRAVERSE :-



Quadrant of closing error will be decided with the help of algebraic sign of e_x & e_y

$$e = \sqrt{e_x^2 + e_y^2}$$

$$\theta = \tan^{-1} \frac{|e_x|}{|e_y|}$$

Que. Coordinate of E is (25, 5) & E' is (20, 1)
Determine Magnitude and direction of closing error.

Soln.

$$E = (25, 5)$$

$$E' = (20, 1)$$

$$e_x = 1 - 5 = -4$$

$$e_y = 20 - 25 = -5$$

$$e = \sqrt{e_x^2 + e_y^2}$$
$$= \sqrt{(-4)^2 + (-5)^2} = \sqrt{41} = 6.403$$

$$\theta = \tan^{-1} \frac{|e_x|}{|e_y|} = \tan^{-1} \frac{1-4}{1-5} = 38.65^\circ$$

$\therefore e_x$ & e_y both are -ve

$\therefore$ closing error lies in IIIrd Quadrant.

$\therefore$ WRB of closing error is 218.65°

$\therefore$ Magnitude of correction is -6.403

Direction of correction will be 38.65°

RELATIVE ERROR OF CLOSE / RELATIVE ACCURACY /
DEGREE OF ACCURACY :-

$$\text{Relative error of close} = \frac{e}{P} = \frac{1}{P/e}$$

Where $e \rightarrow$ Magnitude of closing error

$P \rightarrow$ perimeter of traverse

ADJUSTMENT OF TRAVERSE :-

If there is error of closer then it has to be adjusted such that error of closer becomes 'zero' and traverse is closed geometrically.

1) ARBITRARY METHOD :-

In this method closing error is distributed arbitrary according to the experience of surveyor based on field condition.

2) Bowditch Rule :- / COMPASS RULE :-

It is also called as Compass Rule, it is generally used for adjusting a traverse in which angles and distances are measured with same precision. Bowditch Rule also assumes that error is random in nature.

Therefore probable error in a traverse line is directly proportional to $\sqrt{\text{length of the line}}$

$$\frac{\text{Error in lat or Departure of a line}}{\text{Total error in lat or Dep.}} = \frac{\text{Length of line}}{\text{perimeter of Traverse}}$$

$$e_{L-AB} = e_y \times \frac{AB}{P}$$

→ When traverse is adjusted by this Rule both length & Bearings of the lines are changed.

→ compare to transit Rule length are changed less and angles are changed more.

③ Graphical Method / proportionate Method :-

Graphical Method is based on bowditch Rule
Hence, it is used for compass traverse

④ Transit Rule :-

Transit Rule is used to balance a traverse in which angular measurements are more accurate than linear measurements

Ex: Theodolite Traverse

$$\text{Error in lat or Dep. of a line} = \frac{\text{Total error in lat or Dep.}}{\text{Arithmetic sum of lat. or Departure}} \times \text{Numerical value of lat or Dep of line}$$

In transit rule angles are changed less but length are changed more

* In transit Rule if a line is having zero latitude or zero departure then there will not be any correction to latitude or departure respectively.

Axis - METHOD :-

- In axis method corrections are only applied to length
- Angles are measured precisely and Distances are not measured precisely
- Since Dirⁿ of lines are not affected general shape of traverse is not affected.

GTALES TRAVERSE TABLE :-

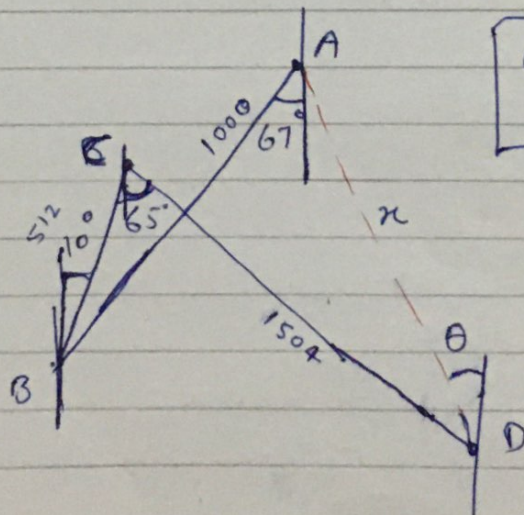
- Traverse calculations are generally done in tabular form which is known as Gales Traverse table
- In this case we ultimately determine independent coordinates of various points after applying necessary correction

It is better to select Independent coordinate such that whole traverse lies in first quadrant and independent coordinate of these points become positive

INVERSING:-

It is calculation of length and bearings of line from end station coordinates.

Que. A surveyor started ^{from} point A and travelled 1000 steps in the dirⁿ S 67° W and reached a point B then he changed his dirⁿ and travelled 512 steps in north 10° E and reached a point C then again he changed his direction and travelled 1504 steps in S 65° E and reached D. Determine how many steps and in which dirⁿ he shall move to reach A.



Quadrantal bearing of
DA = N 45.5° W

Ans.

$$\sum L = -1000 \cos 67^\circ + 512 \cos 10^\circ - 1504 \cos 65^\circ + x \cos \theta = 0$$

$$\sum D = -1000 \sin 67^\circ + 512 \sin 10^\circ + 1504 \sin 65^\circ - x \sin \theta = 0$$

$$x \cos \theta = 522.127$$

$$x \sin \theta = 531.489$$

$$\Rightarrow x = 745 \text{ steps}$$

$$\theta = 45.5^\circ$$

Que. Determine missing readings.

Line	length	bearing
AB	200	Roughly East
BC	98	178°
CD	x	270°
DA	86.4	1°

$$\sum L = 200 \cos \theta + 98 \cos 178^\circ + x \cos 270^\circ + 86.4 \cos 1^\circ = 0$$

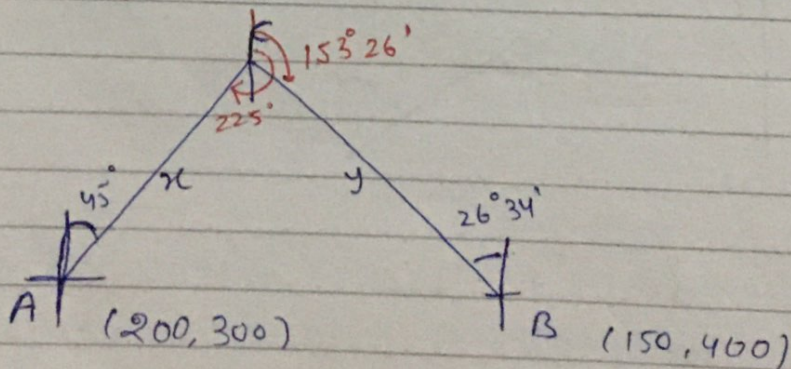
$$\theta = 86.68^\circ$$

$$\sum D = 200 \sin 86.68^\circ + 98 \sin 178^\circ + x \sin 270^\circ + 86.4 \sin 1^\circ = 0$$

$$x = \underline{\underline{204.59 \text{ m}}}$$

Que. Bearing of two inaccessible stations A & B taken from station C were $225^\circ 0'$ & $153^\circ 26'$ respectively. Coordinates of A & B are as given below. Determine coordinate of C.

St ⁿ	N	E
A	200	300
B	150	400



$$200 + x \cos 45^\circ = 150 + y \cos 26^\circ 34'$$

$$300 + x \sin 45^\circ = 400 - y \sin 26^\circ 34'$$

$$x = 70.707, \quad y = 111.802$$

$$L_c = 200 + 70.707 \cos 45^\circ = 250$$

$$D_c = 300 + 70.707 \sin 45^\circ = 350$$

Que: P, Q, R & S are 4 stations. Whose coordinates are as given below.

Stn	N	E
P	1000	1000
Q	1075.18	1180.94
R	1215.62	1021.98
S	1102.36	939.70

Another station X is to be fix at the intersection of lines PR & QS. What are the coordinates of X?

Ans $\rightarrow$ (1094.15, 1009.58)

Ques. Calculate latitude, departure and closing error for the following traverse. And adjust using bowditch Rule.

Line	length	WCB	latitude		
			Calc.	Correction	Corrected
AB	90 m	45°	+63.639	-0.464	+63.175
BC	150 m	300	+75.000	-0.774	+74.226
CA	152 m	154	-136.616	-0.785	
			<u>$\Sigma L = +2.023$</u>	<u>-2.023</u>	<u>+137.401</u>
					<u>0</u>

Departure

calculated	correction	corrected.
+63.639	-0.084	63.555
-129.903	-0.141	-130.044
+66.632	-0.143	66.489
<u>$\Sigma D = +0.368$</u>	<u>-0.368</u>	<u>0</u>

$$e_x = +0.368, e_y = +2.023$$

$$e = \sqrt{e_x^2 + e_y^2} = 2.056$$

$$\theta = \tan^{-1} \frac{|e_x|}{|e_y|} = 10.309^\circ$$

Since e_x & e_y both are positive Hence closing error lies in Ist quadrant
therefor WCB of closing error = 10.309°

$$e_{L-AB} = 2.023 \times \frac{90}{392} = +0.464$$

$$e_{D-AB} = 0.368 \times \frac{90}{392} = +0.084$$

5 LEVELLING

- levelling is the operation of determining height difference of various point
- In topographical work the height are measured with respect to a common frame of reference which is called as 'Datum'
- Generally Datum is Mean level of sea
- Mean level of sea is average height of high and low ~~tide~~ ^{tide} over a long period of time that is 19 year.

BASIC DEFINATIONS:-

① ELEVATION:- RL :- Elevation is the vertical distance of a point above or below a datum.

- Elevation is also called as RL, Height of a point etc.
- Vertical distances are always measured along the dirⁿ of gravity which is plumb line

② Altitude :- Altitude is vertical distance above datum.

- Above datum elevation & Altitude are same

③ Axis of Telescope :- Axis of the telescope is the line joining optical center of objective to the center of eyepies.

③ Line of sight:-

It is line joining intersection of cross hair to the optical center of objective.

④ Line of collimation:-

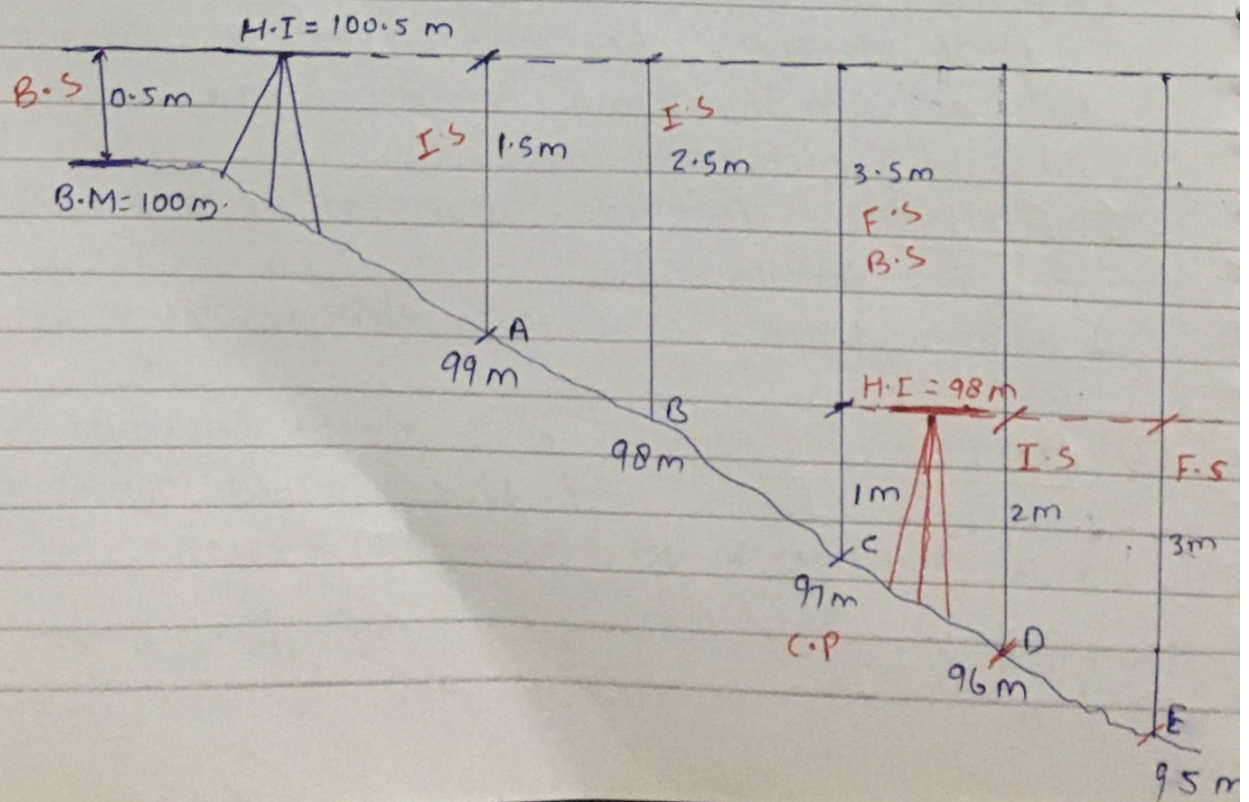
In levelling line of sight has to be horizontal while taking the readings. When line of sight becomes horizontal that is perpendicular to plumb line is called as line of collimation.

⑤ Height of Instrument:-

It is elevation of plane of collimation.

⑥ STATION:- Station is a point of unknown elevation.

Back sight:-



BACK SIGHT:-

Back sight is the first staff reading taken after instrument is set up on the point of known elevation.

- Height of instrument = R.L of benchmark + Back sight
- Back sight is taken on Bench mark for first setting of instrument and thereafter on change point for next setting of instrument.

Fore sight:-

It is the last staff reading taken from a particular setting of instrument

- Fore sight is taken at the last point of survey or at change point.
- Elevation of staff station = H.I - Fore sight

change point / turning point $\Rightarrow$

It is the point denoting shifting of level.

- There are two readings taken at change point one is fore sight from previous setting of instrument and second is back sight from next setting of instrument

Intermediate Sight:-

It is the staff reading taken b/w back sight & fore sight from a particular setting of instrument.

- Both ^{intermediate} ~~Back~~ sight & Fore sight are taken at a point of unknown elevation.
- Fore sight is taken either at change point or last point of survey, whereas, Intermediate sight is taken at any ^{other} point.

Parallax:-

It is apparent movement of image relative to cross hair

BENCH MARK:- It is a fixed reference point of known elevation.

Depending upon permanency and precision bench marks are classified as

1) GTS BENCH MARK:-

Great Trigonometrical survey B.M

- These are established by survey of India throughout the country very accurately with respect to Bombay height

2) permanent B.M:- These are established at closer interval b/w widely spaced GTS B.M by state PWD and survey of India

3) Temporary BENCH MARK :- These B.M are temporarily established whenever required, Generally these are the points at which work is closed for a day and from which work would be started for next day.

4) Arbitrary BENCH MARK :- These are the B.M whose elevation doesn't refer to any fix datum, such as MSL

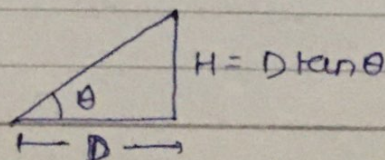
- elevation of such bench mark are arbitrary assumed for levelling of a small Area.

DIFFERENT METHODS OF LEVELLING:-

① DIRECT LEVELLING :- (SPIRIT levelling) :-

- It is most common method of levelling
- A spirit level fixed to the telescope of a levelling instrument is used to make line of sight horizontal.
- vertical distances are measured with respect to horizontal line of sight

② TRIGONOMETRIC LEVELLING :- It is a indirect method of levelling in which difference of elevation is determined indirectly from horizontal distance and vertical angle.



③ BAROMETRIC levelling :- It is another type of indirect levelling in which elevations are determined indirectly from change in atmospheric pressure

- Atmospheric pressure decreases with the increase in elevation.
- Altimeter
- Aneroid barometer and Mercury barometer are used for determining changes in atmospheric pressure
- Altimeter is used for determining altitude of an Aeroplane

④ Hypsometric levelling :- In this process of levelling elevation is determined by observing the temp. at which boiling of water starts

- As Altitude increases boiling point of water decreases

CLASSIFICATION OF Direct levelling :-

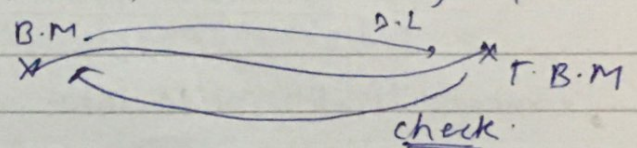
① SIMPLE levelling :- In this method only one setting of instrument is done to determine height difference b/w two points.

② DIFFERENTIAL / COMPOUND levelling :- In this method more than one setting of instrument is required to determine height difference. It is used when

- a) Two points are at large distance.
- b) Difference of elevation b/w two points is large.
- c) obstruction b/w two points.

③ Check levelling:- It is a type of Differential levelling done for the purpose of checking elevation of different points which are already obtained.

Generally check levelling is done at the end of days work from last station to the starting station of that day.



④ FLY LEVELLING:-

It is a type of Differential levelling used for determination of approx. elevation of different points.

- Only back sight and fore sights are taken and line of sight kept as long as possible.
- It is used when accuracy desired is low and time available is less.

⑤ PROFILE LEVELLING / LONGITUDINAL LEVELLING:-

It is a type of differential levelling done for the purpose of determining elevation of ground surface along a fix line.

- Fix line is generally center line of the project.
- It is used for fixing gradient along longitudinal section.

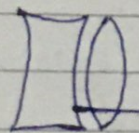
CROSS-SECTIONAL LEVELLING:- It is a type of Differential levelling done to determine difference of elevation of ground surface along the lines perpendicular to center line.

Generally profile levelling and $\frac{1}{2}$ s levelling are done simultaneously for estimation of earthwork.

OPTICAL DEFECT IN LENCE OF TELESCOPE :-

① chrometic abberation:- It occurs in telescope due to dispersion of white light

- White light splits into component of colours which result in formation of rainbow near image due to which focusing become difficult
- This defect is removed by using combination of lences as discussed below



Canada Balsam.

- These two lences are joined together by chemical called as Canada Balsam.

② spherical abberation:-

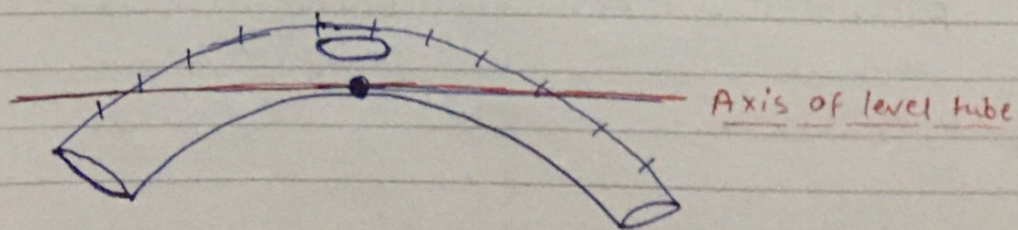
- Spherical abberation occurs due to defect in spherical surface of ~~length~~ lence.
- It is also removed by same combination of lences as discussed above.

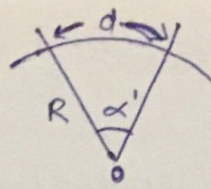
NOTE:- Achrometism $\rightarrow$ absence of chrometic Abberation.
Aplanation $\rightarrow$ absence of spherical Abberation.

LEVEL TUBE:- LEVEL tube or Bubble tube is an essential part of most of the surveying instruments as it is used for making line of sight horizontal and vertical axis coinciding with plumb line.

level tube is a glass tube sealed at both the ends, which is partially filled with a sensitive liquid leaving enough space for the formation of air bubble.

- Liquid should be a) stable & non-freezing at ordinary temperature
b) low viscosity
- previously, Alcohol, ether & chloroform were used but these days purified synthetic alcohol is used.
- C/s of level tube is circular in nature.
- upper surface of level tube of graduation which are symmetrical about center and equally spaced.
- Generally graduations are 2mm long.





$$\alpha' = \frac{d}{R} \times 206265$$

- Longitudinal line tangent to the mid point of inner curved surface is called an axis of level tube.
- During the construction of equipment line of sight is made parallel to axis of level tube.
- When air bubble comes in center axis of level tube becomes horizontal and thereby line of sight becomes line of collimation.

SENSITIVITY OF LEVEL TUBE :- Sensitivity of level tube is expressed in terms of angle in seconds subtended at the center by a arc of level tube = 1 division

- It is expressed as seconds per mm.
- Radius of curvature of upper surface of tube is also a measure of sensitivity.
- If the Radius of curvature of level tube is large is more sensitive.

Bubble Tube A

$$\alpha'_A = 20''/2 \text{ mm}$$

$$\alpha'_A = \frac{d}{R} \times 206265$$

$$R = \frac{2 \times 10^{-3}}{20} \times 206265$$

$$R_A = 20.63 \text{ m}$$

Bubble Tube B

$$\alpha'_B = 10''/2 \text{ mm}$$

$$R_B = \frac{2 \times 10^{-3}}{10} \times 206265$$

$$R_B = 41.25 \text{ m}$$

Here, Bubble tube B is more sensitive because for 10 seconds of deflection Air bubble moves for 2mm whereas in A 20 sec. deflection is required for movement of 2mm.

SENSITIVITY of level tube can be increased by changing following parameter.

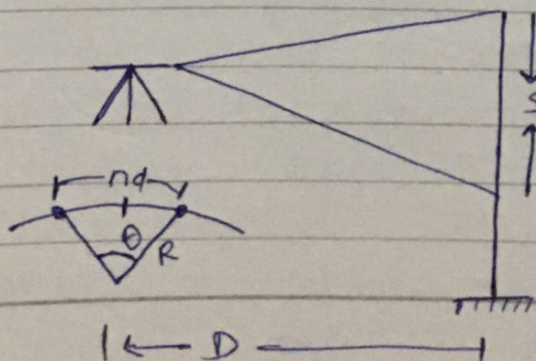
- ① Increasing Radius of curvature of tube
- ② By decreasing Roughness of inner walls of tube
- ③ By reducing Viscosity of liquid
- ④ By increasing diameter of tube.
- ⑤ By increasing length of Air bubble.
- ⑥ By Reducing temp.

NOTE :- SENSITIVITY expressed as sec/division is not a definite quantity unless length of one division is also specified.

• If Not specified then length of one division can be taken as 2mm.

• Length of Air bubble changes with temp and also under the action of gravity.

DETERMINATION OF SENSITIVITY OF BUBBLE TUBE :-



$$\theta = nd/R = \frac{S}{D}$$

$$\alpha' = \frac{d}{R} = \frac{S}{nD} \times 206265$$

Where S , staff intercept

n → number of division moved by Air bubble

D → Horizontal distance b/w vertical axis of instrument & staff station

d → length of one division of ~~air~~ level tube

Que. If Bubble have sensitivity of $20 \text{ sec}/2 \text{ mm}$, determine error in staff reading on a vertically held staff at a distance of 200 m . Caused by 3 divisions out of center.

$$\alpha'' = \frac{S}{n} \times 206265$$

$$20'' = \frac{S}{3 \times \frac{200}{2000}} \times 206265$$

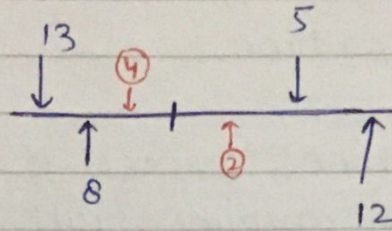
$$S = \frac{20 \times 3 \times 200}{206265}$$

$$S = 0.058 \text{ m}$$

Que. The following observations were made to determine the sensitivity. The distance b/w vertical instrument & staff station is 100 m . length of one division is 2 mm .

Bubble tube	Case	Bubble Reading		Staff reading
		L.H.S	R.H.S	
A	(i)	13	5	1.681
	(ii)	8	12	1.767
B	(i)	15	3	1.635
	(ii)	6	14	1.788

Bubble tube A

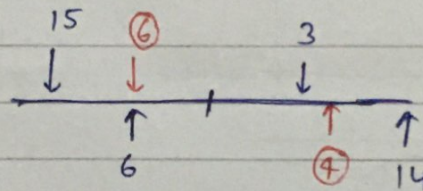


$$n = 4 + 2 = 6$$

$$\alpha'_A = \frac{S}{nD} = \frac{1.767 - 1.681}{6 \times 100} \times 206265$$

$$= 29.56''$$

Bubble tube B



$$n = 10$$

$$\alpha'_B = \frac{1.788 - 1.635}{10 \times 100} \times 206265$$

$$\alpha'_B = 31.55''$$

A is more sensitive.

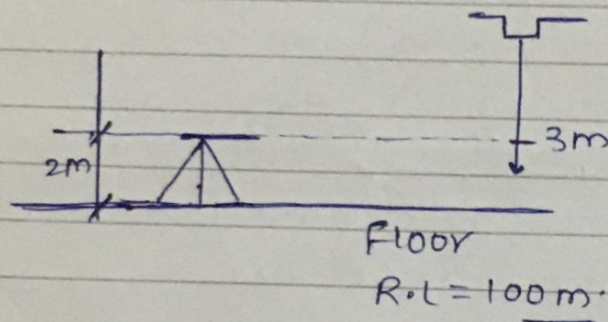
THEORY OF SIMPLE LEVELLING :- • level is set up at a suitable place b/w

Two stations

- Instrument need not to be in line with end stations.
- RL of Ground point of Instrument station is not determined.
- Staff should be vertical & line of sight should be horizontal
- To ensure verticality of staff, staff is waved slightly towards instrument and then away from instrument. This process is called as 'Rocking' of staff.

- During this process instrument man should observe smallest reading
- Very intense sunshine causes Air close to earth to shimmer Hence, avoid staff reading less than 0.5 m.

USE OF INVERTED STAFF :-



$$\begin{aligned}
 \text{R.L of soffit of beam} &= \\
 &= (100 + 2) - (-3) \\
 &= \underline{\underline{105 \text{ m}}}
 \end{aligned}$$

When the point whose elevation has to be determined is much above the line of sight example $\Rightarrow$ soffit of beam or soffit of slab, then to determine level of that point staff is kept inverted with zero m end touching that point

- Such readings are recorded with a -ve sign in level field book.

level Field Book :-

A level field book is use to record ~~and~~ reduce level of various points. There are two methods of reducing the levels.

$$R.L = H.I - \text{Reading}$$

$$\underline{H.I = R.L + B.S}$$

- i) Height of Instrument Method / collimation Method
- ii) Rise and fall Method

Que. The following conjugative readings were taken on a continuous sloping ground with a 4m step if reduced level of B.M was 100 m then determine R.L of all the other points by both the Methods.

	St ⁿ	B.S	I.S	F.S	H.I	R.L	R.M
0.780	A	0.780	-	-	100.780	100	B.M
1.535	B	-	1.535	-	-	99.245	
2.985	C	-	1.955	-	-	98.825	
3.480	D	-	2.985	-	-	97.795	
1.155	E	1.155	-	3.480	98.455	97.300	CP
2.365	F	-	2.365	-	-	96.090	
3.640	G	0.935	-	3.640	95.750	94.815	CP
0.935	H	-	1.630	-	-	94.120	
1.630	I	-	-	2.545	-	93.205	Last point
2.545		<u>2.870</u>		<u>9.665</u>			

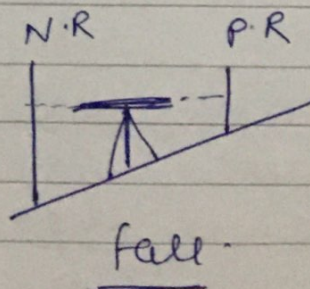
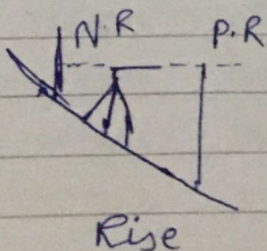
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Airthmetic check :- $\sum B.S - \sum F.S = \text{Last R.L} - I^{\text{st}} \text{ R.L}$

$$2.870 - 9.665 = 93.205 - 100$$

$$- 6.795 = - 6.795$$

Rise - fall Method :-



0.780	St ⁿ	B.S	I.S	F.S	Rise	Fall	R.L	R.M
1.535	A	0.780	-	-	-	0.755	¹⁰⁰ 99.245	B.M
1.955	B	-	1.535	-	-	0.755	99.245	
2.985	C	-	1.955	-	-	0.420	98.825	
3.480	D	-	2.985	-	-	1.03	97.795	
1.155	E	1.155	-	3.480	-	0.495	97.3	C.P
2.365	F	-	2.365	-	-	1.210	96.090	
3.640	G	0.935	-	3.640	-	1.275	94.815	C.P
0.935	H	-	1.630	-	-	0.695	94.120	
1.630	I	-	-	2.545	-	0.915	93.205	last point
2.545		2.870		9.665	0	6.795		

CHECK $\sum B.S - \sum F.S = \text{last R.L} - \text{1^{st}} R.L = \sum Rise - \sum Fall$

$-6.795 = -6.795 = -6.795$

Rise & fall Method is better than Height of Instrument Method because

- 1) It gives better visualization of terrain
- 2) R.L of I.S are also checked.

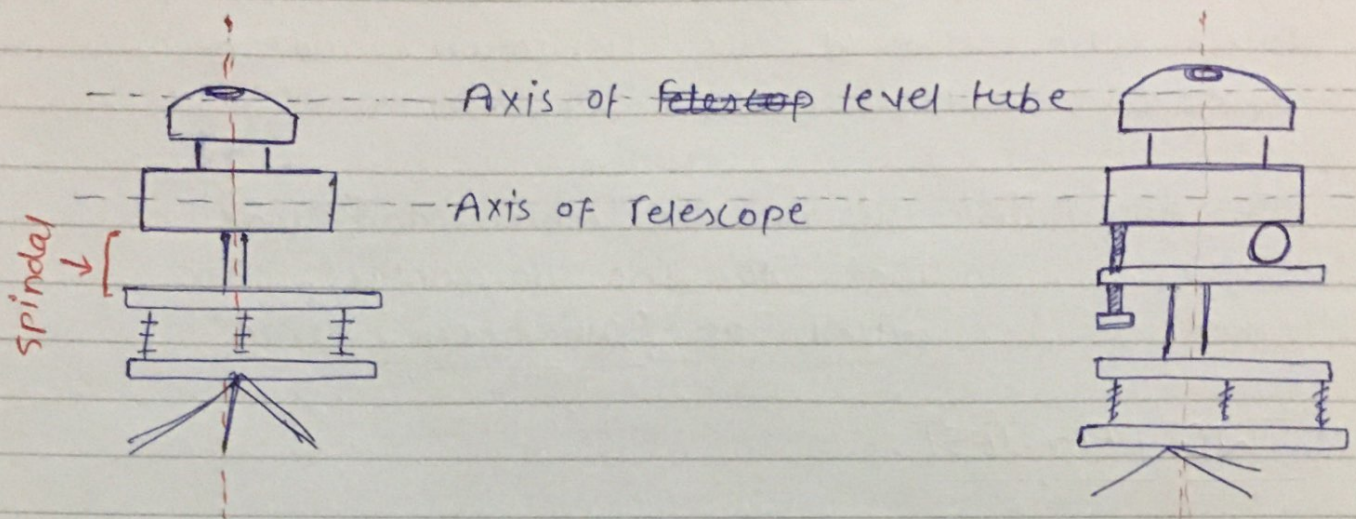
Que: Fill the missing Quantities and apply usual check

St ⁿ	B.S	I.S	F.S	Rise	Fall	R.L	R.M
1	3.125					124.180	B.M
2	2.265		1.800	1.325		125.505	F.P
3		2.320			0.055	125.450	
4		1.920		0.400		125.850	
5	1.040		2.655		0.735	125.115	F.P
6	1.620		3.205		2.165	122.950	T.P
7		3.625			2.005	120.945	
8			1.480	2.145	-	123.090	F.B.M
	8.050		9.140	3.870	4.960		

$\sum B.S - \sum F.S = \text{last R.L} - \text{1^{st}} R.L = \sum Rise - \sum Fall$

$-1.090 = -1.090 = -1.090$

DUMPY LEVEL:-



In Dumpy level Telescope and spindal are cast as one piece

- levelling is done using levelling screws
- Under proper adjustment axis of level tube should be perpendicular to vertical axis of Instrument

In Tilting level this adjustment is not required.

PERMANENT ADJUSTMENT OF DUMPY LEVEL:-

- (i) Axis of level tube should be parallel to line of sight
- (ii) Axis of level tube should be perpendicular to vertical axis of Instrument

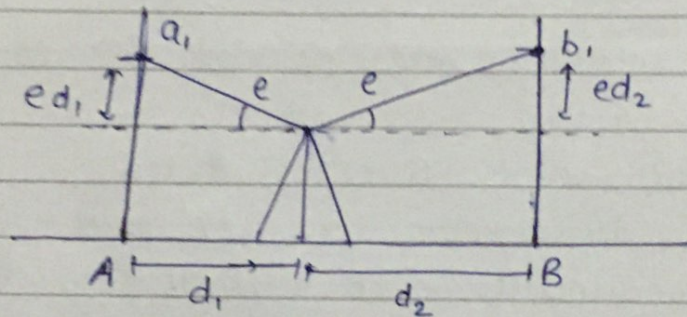
This adjustment is not required in Tilting level

- (iii) Horizontal crosshair should be in a plane perpendicular to vertical axis of Instrument.

When Air Bubble is in the center Axis of level tube should be Horizontal and also line of sight should be Horizontal

If Air Bubble is in the center and line of sight is inclined to the Horizontal plane then it is called as 'collimation error'

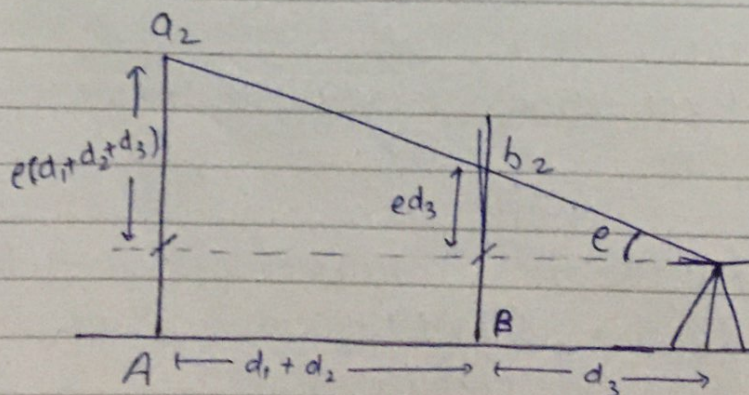
TWO PEG TEST:-



$$\Delta h_{AB} = (a_1 - ed_1) - (b_1 - ed_2)$$

if $d_1 = d_2$

$$\Delta h_{AB} = a_1 - b_1$$



$$\Delta h_{AB} = (a_2 - e(d_1 + d_2 + d_3)) - (b_2 - ed_3)$$

$$\Delta h_{AB} = (a_2 - b_2) - e(d_1 + d_2)$$

$$(a_1 - b_1) = (a_2 - b_2) - e(d_1 + d_2)$$

$$e = \frac{(a_2 - b_2) - (a_1 - b_1)}{d_1 + d_2}$$

Que: In order to check adjustment of a dumpy level the following procedure was followed.

A, B, C & D were selected in a straight line such that $AB = BC = CD = 30$ m.

Determine whether the instrument has collimation error or not if yes then how much?

<u>Inst</u>	staff reading	
St ⁿ	A	C
B	1.926	1.462
D	2.445	1.945

Instrument station B

$$\Delta h_{AC} = 1.926 - 1.462 = 0.464$$

Instrument station D

$$\Delta h_{AC} = 2.445 - 1.945 = 0.5$$

Since Δh_{AC} is not equal in both the cases Hence, there is collimation error

$$e = \frac{0.5 - 0.464}{60} = 0.0006 \text{ m/m}$$

EFFECT OF CURVATURE:-

Effect of curvature occurs because difference b/w level line and horizontal line increases as the distance of staff station from instrument station increases.

level line is a curved line whereas line of collimation is a horizontal line which is tangential to level line

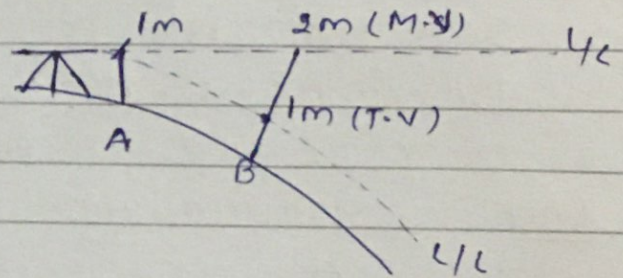
therefore points on the earth surface appears to be much lower than actually they are.

$$M.V > I.V$$

$$E = +ve$$

$$C_c = -ve$$

$$C_c = -0.0785 d^2$$



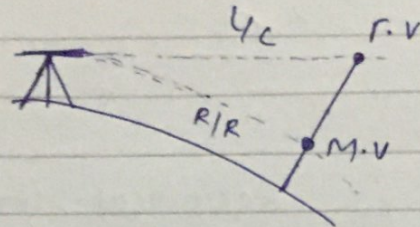
Where $C_c \rightarrow$ Curvature correction in (m)
 $d \rightarrow$ Horizontal distance b/w instrument and staff station in (km)

Refraction correction:-

Density of Air in the atmosphere decreases with the increase in altitude, Air is denser near the earth.

A Ray of Light Travels from thinner Medium to Denser Medium therefore It bends towards the Normal.

Hence, line of sight does not remain horizontal but it bends downwards towards the center of the earth.



$$M.V. < F.V$$

$$E_r = -ve$$

$$C_r = +ve$$

$$C_r = \frac{1}{7} |C_c| = +0.0112d^2$$

COMBINED CORRECTION:

$$C = C_c + C_r$$

$$C = -0.0785d^2 + 0.0112d^2$$

$$C = -0.0673d^2$$

$$d \rightarrow \text{KM}$$

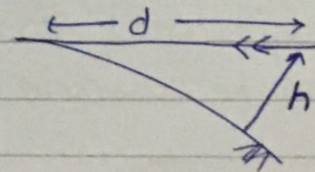
$$C \rightarrow \text{m}$$

In ordinary survey effect of curvature & refraction are neglected for a distance less than 250 m.

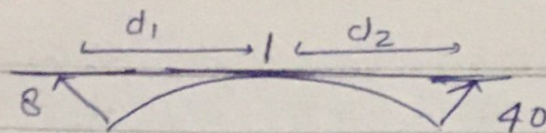
DISTANCE OF VISIBLE HORIZON:-

$$h = 0.0673d^2$$

$$d = 3.855\sqrt{h}$$



Que · An observer standing on the deck of a ship just sees the light house which is 40m above the sea level. If Height of observers eye is 8m above the sea level then Determine the distance b/w observer & light house.



$$d_1 + d_2 = 3.855 (\sqrt{8} + \sqrt{40})$$

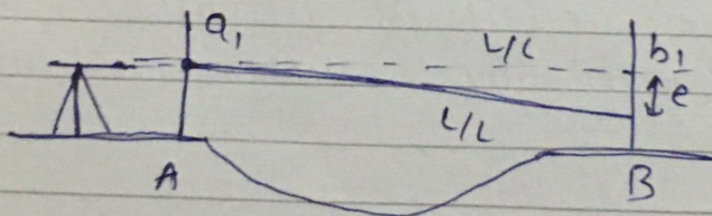
$$= 35.28 \text{ km.}$$

RECIPROCAL LEVELLING:-

Reciprocal levelling is used to determine correct difference in elevation of two points which are quite a large distance apart and it is not possible to set up the instrument midway b/w these points.

Ex. River (opposite banks) or opposite side of valley.

- Reciprocal levelling eliminates error due to
1) Curvature 2) Refraction 3) Collimation



e is total error which is due to collimation, refraction and curvature

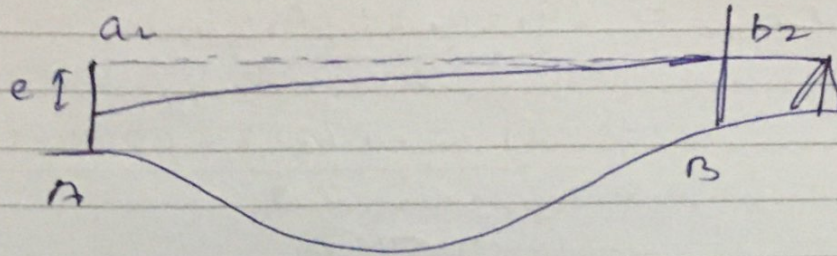
$$e = e_l + e_c + e_r$$

e_l → collimation error assumed as inclined upward and positive

e_c → curvature error, +ve

e_r → refraction error → -ve

Assuming stⁿ A is lower than stⁿ B
 True $\Delta h_{AB} = a_1 - (b_1 - e) \quad \text{--- (1)}$



$$\Delta h_{AB} = (a_2 - e) - b_2 \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2} \quad 2\Delta h_{AB} = (a_1 - b_1) + (a_2 - b_2)$$

$$\Delta h_{AB} = \frac{(a_1 - b_1) + (a_2 - b_2)}{2}$$

CALCULATION FOR COLLIMATION ERROR :-

$$e = e_1 + e_c + e_r$$

$$e_1 = e - e_c - e_r$$

$$\Delta h_{AB} = a_1 - (b_1 - e)$$

Que. The following readings refer to reciprocal levels taken with one instrument.

Inst.	staff reading		R.M
st ⁿ	A	B	
A	1.03	1.630	AB = 800 m
B	0.950	1.540	R.L of A = 100 m

Determine True RL of B. Combined Correction and Collimation error.

Assuming A is lower than B

$$R.L \text{ of } B = R.L \text{ of } A + \Delta h_{AB}$$

$$\begin{aligned}\Delta h_{AB} &= \frac{(a_1 - b_1) + (a_2 - b_2)}{2} \\ &= \frac{(1.03 - 1.630) + (0.95 - 1.54)}{2} \\ &= -0.595 \text{ m}\end{aligned}$$

NOTE A is higher than B.

$$R.L \text{ of } B = 100 + (-0.595)$$

$$R.L \text{ of } B = 99.405 \text{ m}$$

$$C = -0.0673 d^2 = -0.043 \text{ m}$$

Calculation for collimation error:-

$$e = e_1 + e_c + e_r$$

$$\Delta h_{AB} = a_1 - (b_1 - e)$$

$$-0.595 = 1.03 - (1.630 - e)$$

$$e = +0.005$$

$$\Delta h_{AB} = (a_2 - e) - b_2$$

$$-0.595 = (0.950 - e) - 1.540$$

$$e = +0.005$$

$$e_1 = e - (e_c + e_r)$$

$$e_1 = e + C$$

$$e_1 = 0.005 - 0.043$$

$$\underline{\underline{e_1 = -0.038 \text{ m}}}$$

Since ~~the~~ e_1 is -ve Hence line of collimation is inclined downward.

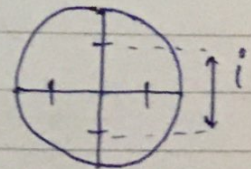
[7.] TACHEOMETRY

- Tacheometry is a branch of surveying in which horizontal and vertical distances are measured with the help of instrument called as tacheometer.
- It is also called as 'optical Distance measurement' Method.
- Tacheometry is extremely useful for rough terrain that is when chaining would be difficult.
- As compared to chaining on flat ground accuracy of tacheometer is low. But on rough terrain and steep ground accuracy is more.

TACHEOMETER :-

It is essentially a transit theodolite, the diagram of which is furnished with stadia wire in addition to cross-wire.

Stadia Hairs are equidistant from central cross hair. This distance is called as stadia interval. (i)



Imp. characteristics of tacheometer :-

- ① Multiplying constant $K = 100$
- ② Additive constant $C = 0$
- ③ Telescope is fitted with analytic lens.

④ Magnifying power of eyepis are kept high

Stadia Rod :-

It is a vertical staff, which is used when distance b/w telescope and staff is so large such that graduations become indistinct in ordinary staff.

- Beyond 100 m distance stadia rod is used

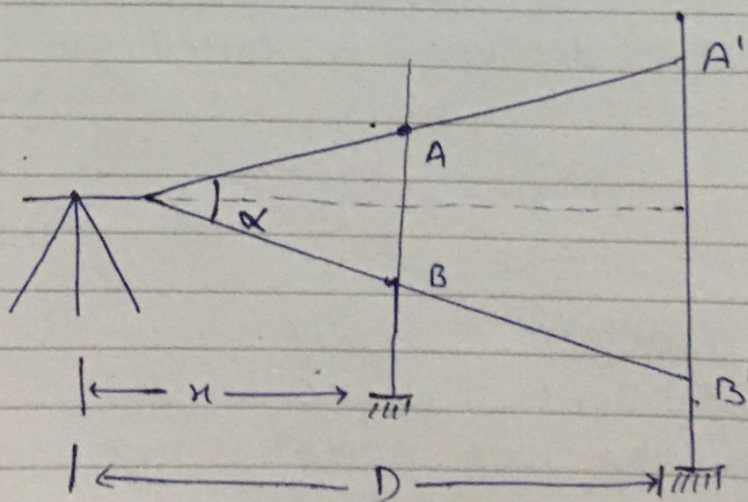
Subtense BAR :-

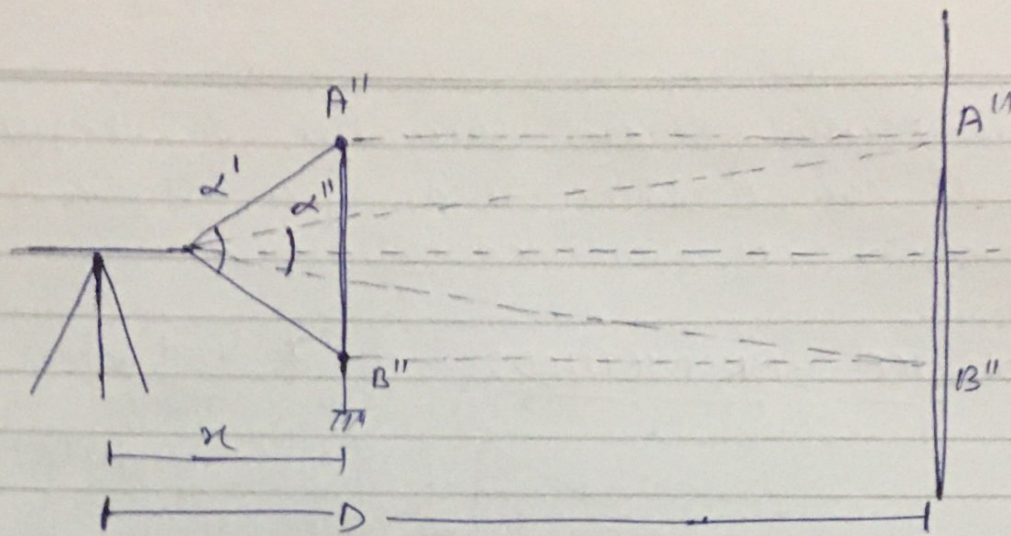
It is a horizontal staff used for distance measurement.

METHODS OF Tacheometry :-

① STADIA METHOD :- In this method stadia intersect (b/w upper & lower stadia Hair) and parallactic angle α is measured.

a) Fix Hair Method →





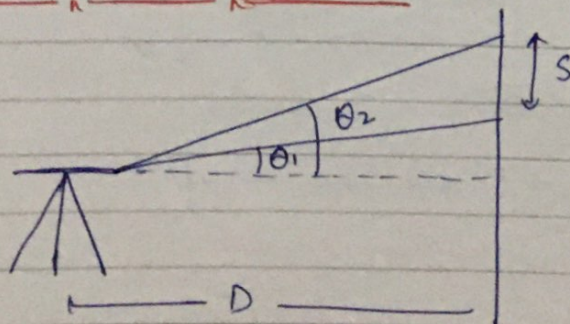
Fixed hair method → In this method vertical spacing b/w upper stadia & lower stadia hair called stadia interval is kept fixed.

In other words paralytic angle α defined with the help of stadia wire is kept fixed and staff ~~to~~ intercept is varied.

Movable Hair Method → In this method staff interval is kept fixed & stadia interval is varied using micrometer screw.

NOTE → As it is inconvenient to measure stadia hair interval accurately, movable hair method is rarely used.

② TANGENTIAL METHOD:-

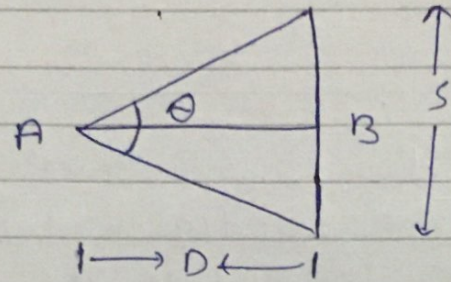


$$S = D(\tan \theta_2 - \tan \theta_1)$$

In this Method stadia hairs are not required
Hence it can also be used when stadia
diagram is not provided

(3) SUBSENCE BAR METHOD:-

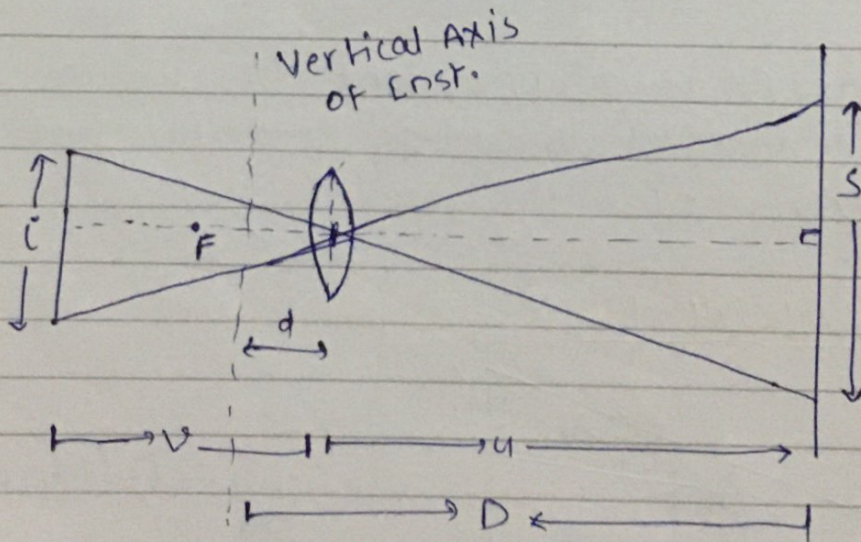
* used when
ground is
undulating.



A Horizontal staff of fix length is placed
at strⁿ B. Horizontal distance D can be
determined with the help of Horizontal
angle θ and staff intersect S

PRINCIPLE OF STADIA METHOD:-

Let us assume that line of sight is horizontal
and staff is held vertical.



$$D = u + d$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$D = \left(\frac{f}{i}\right) s + (f + d)$$

$$\boxed{D = Ks + c} \text{ Tacheometric Distance eqn}$$

Where D = Horizontal distance b/w verticle axis of instrument & staff station

$f \rightarrow$ focal length of objective

$i \rightarrow$ stadia interval

$K \rightarrow$ Multiplying constant or stadia interval factor, for ease in calculation it's value is kept 100.

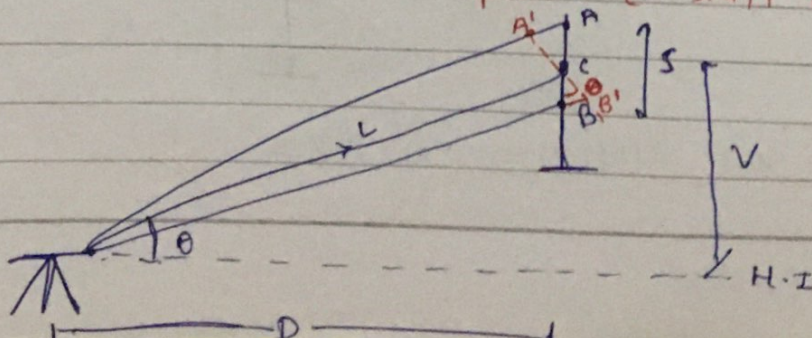
$c \rightarrow$ Additive constant, it's value ranges b/w 0.25 - 0.35 m

if telescope is fitted with anallitic lence then value of c becomes zero

When c is 0 then $D \propto s$

Distance and elevation Formula for inclined line of sight:

① Line of sight inclined upward & staff is vertical.



$$D = L \cos \theta$$

$$V = L \sin \theta$$

$$L = KA'B' + c$$

$$A'B' = A'c + B'c$$

$$A'c = AC \cos \theta$$

$$B'c = BC \cos \theta$$

$$A'B' = AC \cos \theta + BC \cos \theta$$

$$A'B' = (AC + BC) \cos \theta$$

$$A'B' = S \cos \theta$$

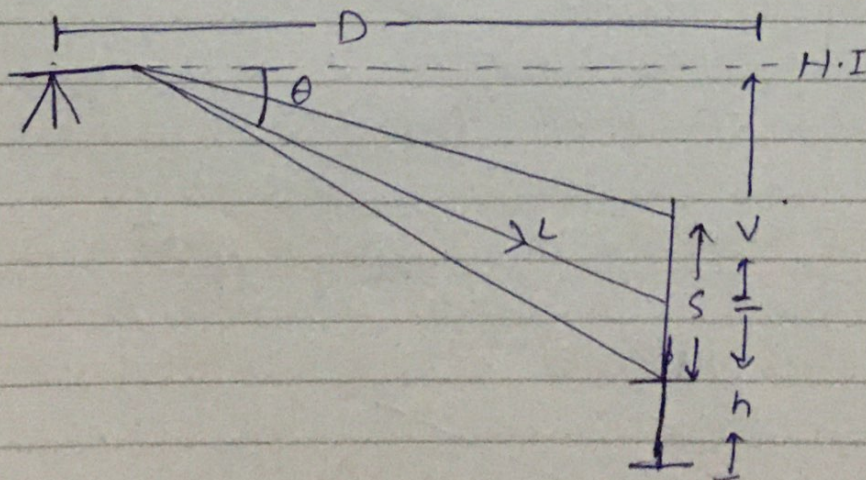
$$L = KS \cos \theta + c$$

$$D = (KS \cos \theta + c) \cos \theta$$

$$V = (KS \cos \theta + c) \sin \theta = \frac{1}{2} KS \sin 2\theta + c \sin \theta$$

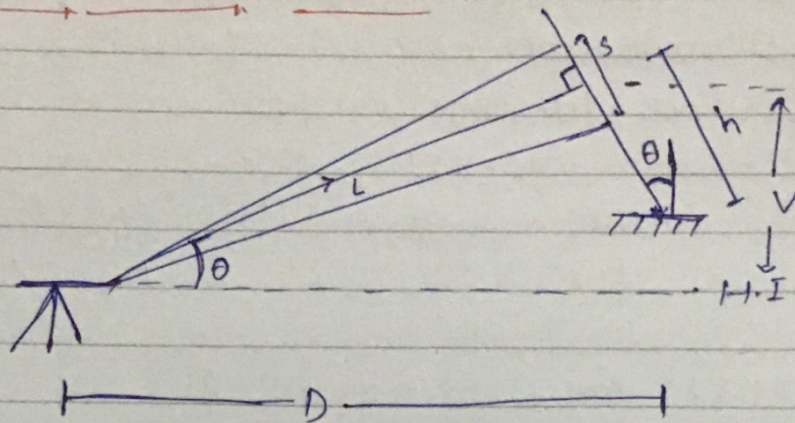
$$\text{Elevation of staff station} = H.I + V - h$$

Case-2. (2) Line of sight is inclined downward and staff is held vertical:-



$$\text{El. of staff station} = H.I - V - h$$

case III (3) Line of sight is inclined upward and staff is held Normal



$$L = Ks + C$$

$$D = L \cos \theta + h \sin \theta$$

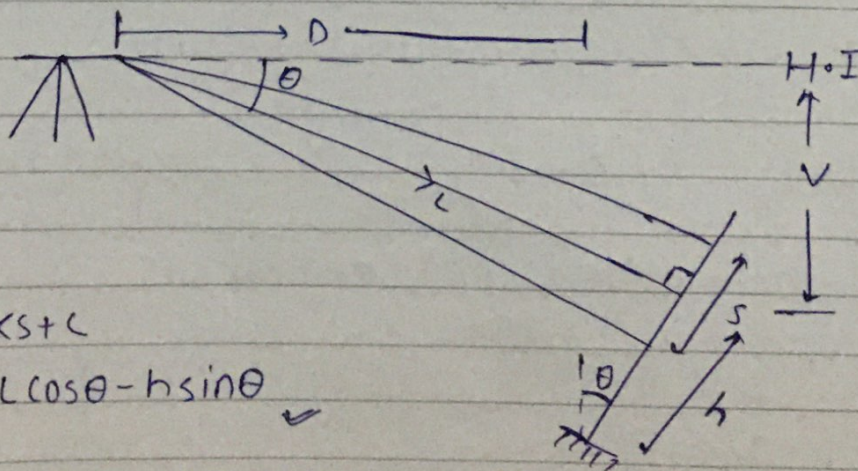
$$\boxed{D = (Ks + C) \cos \theta + h \sin \theta}$$

$$V = L \sin \theta$$

$$V = (Ks + C) \sin \theta$$

$$\text{El. of staff station} = H.I + V - h \cos \theta$$

case - IV (4) Line of sight is inclined Downward and staff is held Normal



$$L = Ks + C$$

$$D = L \cos \theta - h \sin \theta$$

$$\text{El. of S.S} = H.I - V - h \cos \theta$$

Que Determine the gradient from point P to another point Q with the help of following observations
 if $K=100$ and telescope is fitted with Anallatic lens
 Ins. stⁿ staff stⁿ Bearing Vertical Angle staff reading.

P	130°	+ 10° 32'	1.255, 1.81, 2.365	
R	Q	190°	+ 5° 6'	1.3, 2.12, 2.94

Instrument stⁿ ~~at~~ R staff stⁿ P

$$PR = Ks \cos^2 \theta = 100 \times (2.365 - 1.255) \cos^2 10^\circ 32'$$

$$= 107.29 \text{ m.}$$

$$V_{PR} = \frac{1}{2} Ks \sin 2\theta = \frac{100}{2} \times (2.365 - 1.255) \sin 21^\circ 4'$$

$$= 19.95 \text{ m.}$$

$$R.L \text{ of } P = H.I + 19.95 - 1.81$$

Similarly Ins. stⁿ R staff stⁿ Q

$$QR = 100 (2.94 - 1.3) \cos^2 5^\circ 6'$$

$$= 162.6704 \text{ m.}$$

$$V_{QR} = \frac{100}{2} \times (2.94 - 1.3) \sin 10^\circ 12'$$

$$= 14.521 \text{ m.}$$

$$R.L \text{ of } Q = H.I + 14.521 - 2.12$$

$$V_{PQ} = R.L \text{ of } Q - R.L \text{ of } P = 5.739 \text{ m.}$$

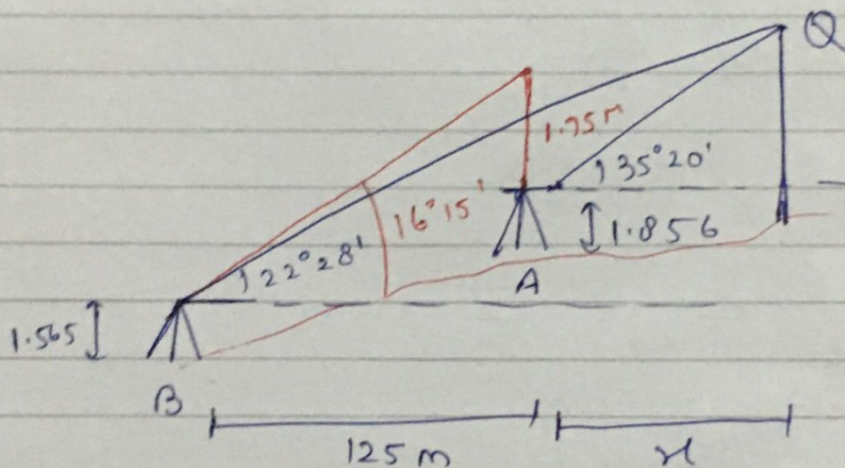
$$PQ^2 = PR^2 + QR^2 - 2 (PR) (QR) \cos 60^\circ$$

$$PQ = 143.27$$

$$G = \frac{V_{PQ}}{PQ} = \frac{1}{25}$$

TRIGONOMETRIC LEVELLING

Que: The top of stag was sighted from 2 stⁿ A & B which are 125 m apart and are in same vertical plane with the top of stag. Angle of elevation of the top of stag observed from instrument stⁿ A is $35^{\circ}20'$ and that observed from instrument stⁿ B is $22^{\circ}28'$. Angle of elevation from stⁿ B to a vane of 1.75 m above the foot of staff held at stⁿ A is $16^{\circ}15'$. Instrument height at stⁿ A & B are 1.856 m and 1.565 m respectively. R.L of stⁿ B is 100. Determine R.L of top of stag.



$$\text{R.L of } Q = \text{R.L of } A + 1.856 + x \tan 35^{\circ}20'$$

$$\text{R.L of } Q = \text{R.L of } B + 1.565 + (x + 125) \tan 22^{\circ}28'$$

$$\begin{aligned} \text{R.L of } A &= \text{R.L of } B + 1.565 + 125 \tan 16^{\circ}15' - 1.75 \\ &= 136.611 \text{ m} \end{aligned}$$

$$136.611 + 1.856 + x \tan 35^{\circ}20' = 100 + 1.565 (x + 125) \tan 22^{\circ}28'$$

$$x = 51.294 \text{ m}$$

$$\begin{aligned} \text{R.L of } Q &= 136.25 + 1.856 + 51.294 \tan 35^{\circ}20' \\ &= 174.466 \text{ m} \end{aligned}$$

Que. Determine the R.L of a church spire C from the following observations from 2 stⁿs A & B which are 50 m apart

$$\angle BAC = 60^\circ, \angle ABC = 50^\circ$$

Angle of elevation from A to top of spire = 30°

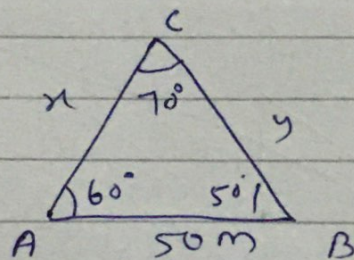
~~29°~~ Angle of elevation from B to top of spire = 29°

Staff reading from A to a B.M of R.L 20 m = 2.5 m.

Staff reading from B to B.M = 0.5 m.

$$\text{R.L of C} = 22.5 + x \tan 30^\circ = 46.032 \text{ m}$$

$$\text{R.L of C} = 20.5 + y \tan 29^\circ = 46.042 \text{ m.}$$



$$\frac{x}{\sin 50^\circ} = \frac{y}{\sin 60^\circ} = \frac{50}{\sin 70^\circ}$$

$$x = 40.760 \text{ m}$$

$$y = 46.080 \text{ m.}$$

$$\text{avg. R.L of C} = \frac{46.032 + 46.042}{2}$$

$$\boxed{\text{R.L of C} = 46.037 \text{ m}}$$

MEASUREMENT OF AREA & VOLUME

One of the major objective of surveying is to determine relative quantities like Area & Volume

AREA :-

1) Field MEASUREMENT:-

If the shape of tract is enclosed by straight boundaries, It can be subdivided into simple geometrical shapes such as Triangle, Trapezoid, Rectangle etc.

The Area of these figure can be determine by using appropriate Formulas.

If the boundaries are irregular than approximate Methods can be used.

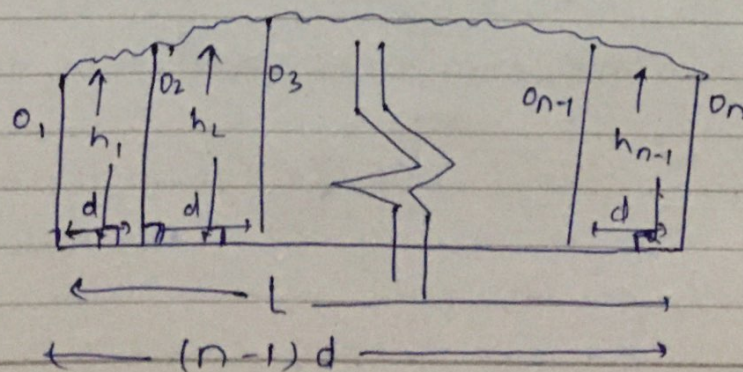
2) plan Measurement :-

Area can also be determined based on the measurements scaled from the plan.

Scale from the plane or by use of planimeter

NOTE planimeter is use to measure the area of a irregular shape on the plan

Approximate METHODS :-



① MED. ORDINATE RULE:-

In this method perpendicular offsets are measured at the mid point of each secⁿ of irregular boundaries.

$$A = \frac{h_1 + h_2 + \dots + h_{n-1}}{n-1} \times L$$

$$A = (h_1 + h_2 + \dots + h_{n-1}) d$$

② AVERAGE ORDINATE RULE:-

In this method perpendicular offsets $o_1, o_2, \dots, o_n$ are measured at the end point of each section towards irregular boundary from the straight line

$$A = \frac{o_1 + o_2 + o_3 + \dots + o_{n-1} + o_n}{n} \times L$$

③ Trapezoidal Rule:-

In this method each section is considered as a trapezoid

$$A = d \left[\frac{o_1 + o_n}{2} + o_2 + o_3 + \dots + o_{n-1} \right]$$

• End ordinates are considered even they are zero

④ SIMSON'S 1/3rd RULE:- If irregular boundaries are curved, this Rule is preferred over Trapezoidal Rule.

This Rule assumes that short length of boundaries b/w two adjacent ordinate is parabolic in nature.

$$A = \frac{d}{3} \left[O_1 + O_n + 4(O_2 + O_4 + O_6 + \dots) + 2(O_3 + O_5 + \dots) \right]$$

* The Number of offset has to be Odd.

When number of offsets are even then simson's Rule can be applied, In such a case Total Area equal to $A_1 + A_2$

A_1 = Area till 2nd last offset by simson's $\frac{1}{3}$ Rule

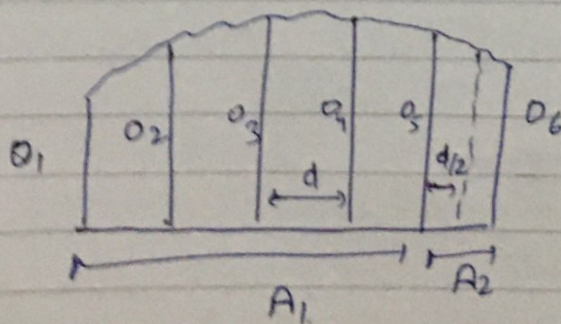
A_2 = Area b/w last 2 offsets by Trapezoidal Rule

if only simson's Rule has to be applied then

Total Area $A = A_1 + A_2$

A_1 = Area till 2nd last offset by simson's $\frac{1}{3}$ rd Rule

A_2 = Area We'll assume an offset O_m as the avg. offset at the mid point of section such that Number of offset becomes odd in Number and Hence, simson's $\frac{1}{3}$ rd Rule can be applied.



$$O_m = \frac{O_5 + O_6}{2}$$

$$\frac{d}{6} \left[(O_5 + O_6) + 2(O_5 + O_6) \right]$$

Que · following perpendicular offsets were taken at 5 m interval from a traverse line towards an irregular boundaries.

2.10, 2.23, 2.80, 3.16, 2.56

Determine the Area by Average Ordinate Rule, Trapezoidal Rule & Simpson's $1/3^{\text{rd}}$ Rule.

Solⁿ. 1) $A = \frac{2.10 + 2.23 + 2.80 + 3.16 + 2.56}{5} \times 20$

$$A = 51.4 \text{ m}^2$$

2) $A = \left[\frac{2.10 + 2.56}{2} + 2.23 + 2.80 + 3.16 \right] \times 5$

$$A = 52.6 \text{ m}^2$$

3) $A = \frac{5}{3} \left[\frac{2.10 + 2.56}{2} + 4(2.23 + 3.16) + 2(2.80) \right]$

$$A = 53.03 \text{ m}^2$$

Que · following perpendicular offsets were measured from a straight line towards an irregular boundary. Determine the Area by Trapezoidal Rule.

Ch (m)	offset (m)
0	1.1
10	1.3
20	0.9
30	0.8
40	1.2
50	1.4
55	1.5
60	1.6

$$d = 10 \text{ m}, 5 \text{ m}$$

$$A = 10 \left[\frac{1.1 + 1.4}{2} + 1.3 + 0.9 + 0.8 + 1.2 \right] + 5 \left[\frac{1.4 + 1.6}{2} + 1.5 \right] = 69.5 \text{ m}^2$$

MEASUREMENT OF VOLUME :-

① END AREA RULE :-

$$V = d \left[\frac{A_1 + A_n}{2} + A_2 + A_3 + \dots + A_{n-1} \right]$$

② PRISMOIDAL RULE :-

$$V = \frac{d}{3} \left[A_1 + A_n + 4(A_2 + A_4 + \dots) + 2(A_3 + A_5 + \dots) \right]$$

For prismoidal Rule Number of c/s has to be ODD.

if Number of c/s are even then total volume

$$V = V_1 + V_2$$

$V_1 \rightarrow$ Volume till 2nd last ~~offset~~ c/s by prismoidal Rule

$V_2 \rightarrow$ Volume b/w last 2 c/s by End Area Rule.

if Only prismoidal Rule is to be used then

$$\text{Total Volume } V = V_1 + V_2$$

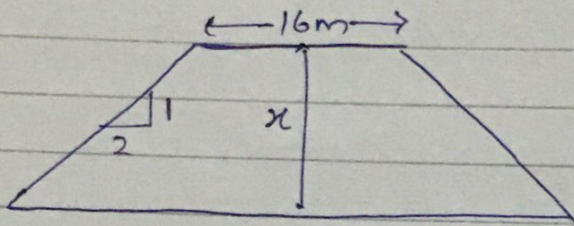
$V_1 \rightarrow$ Volume till 2nd last c/s

$V_2 \rightarrow$ Volume b/w last 2 c/s can be determined

by calculating Area Half Way b/w the end section
interpolating by averaging the sides and not by
averaging the end areas.

Side slope $\rightarrow H:V$
 longitudinal slope $\rightarrow V:H$

Que. A Railway Embankment is 16 m wide, With a its side slope 2:1. Assume Ground to be levelled in the dirⁿ of Traverse line. Calculate volume by END Area Rule in a length of 100 m if center height at 20 m intervals are as given below:-
 2.0, 4.5, 4.0, 3.5, 2.5, 1.5



$$A_x = (16 + 2x)x$$

$$A_1 = 40 \text{ m}^2$$

$$A_2 = 112.5 \text{ m}^2$$

$$A_3 = 96 \text{ m}^2$$

$$A_4 = 80.5 \text{ m}^2$$

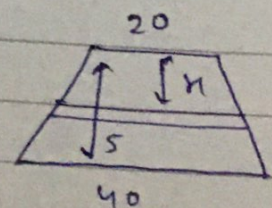
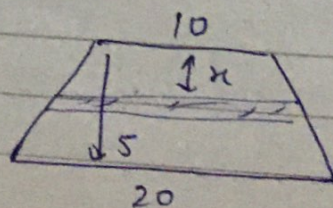
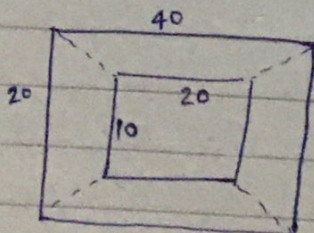
$$A_5 = 52.5 \text{ m}^2$$

$$A_6 = 28.5 \text{ m}^2$$

$$V = 20 \left[\frac{40 + 28.5}{2} + 112.5 + 96 + 80.5 + 52.5 \right]$$

$$V = 7515 \text{ m}^3$$

Que. Determine Volume of a 5m deep pit whose top & bottom Dimensions are 10m x 20m and 20m x 40m.



$$= \int_0^5 \left(10 + \frac{(20-10)}{5} x \right) \left(20 + \frac{(40-20)}{5} x \right) dx$$

$$= 2333.33 \text{ m}^3$$

$$A_1 = 40 \times 20 = 800 \text{ m}^2$$

$$A_2 = 20 \times 10 = 200 \text{ m}^2$$

$$A_m = \frac{800 + 200}{2} = 500 \text{ m}^2$$

$$d = 2.5 \text{ m.}$$

$$V = \frac{2.5}{3} \times [(800 + 200) + 4 \times 500] = 2500 \text{ m}^3$$

$$= 2333.33 \text{ m}^3 \leftarrow$$

avoid this

$$V = \frac{h}{3} \times (A_1 + A_2 + \sqrt{A_1 A_2})$$

↓

Frustum of a pyramid (But only when c.h is same)

PHOTOGRAMMETRY

photogrammetry is the science of obtaining info. about physical objects through the process of recording, measuring and interpretation of photographs.

photogrammetry can be classified as

ARIAL

- photographs of the Area are taken with a camera mounted on an Aircraft
- Camera axis is nearly vertical
- It is used for topographical or preliminary survey.
Ex: Forest, Agriculture pipe line, etc

TERRESTRIAL

- photographs of the Area are taken by camera fixed on or near the ground surface.
- nearly horizontal
- for plotting or study of special features
Ex: vertical cliff or mountain terrain

ADVANTAGES OF ARIAL PHOTOGRAMMETRY:-

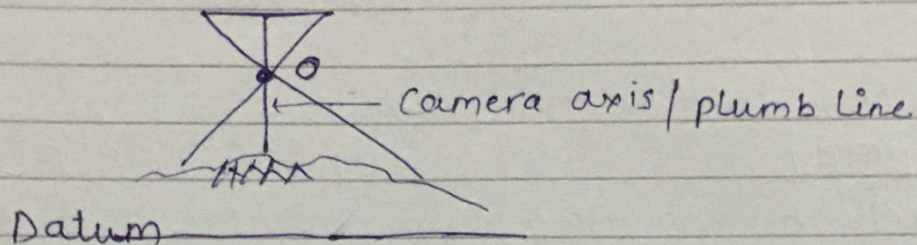
- It is quick
- photographic mapping can be easily done for inaccessible Area and in adverse climate.
- There is no possibility of missing any detail

Disadvantages:-

- For small Area Aerial photogrammetry is costly than ground methods
- It requires costly equipments.
- contour lines can't be easily located.

Types of Aerial Photographs :-

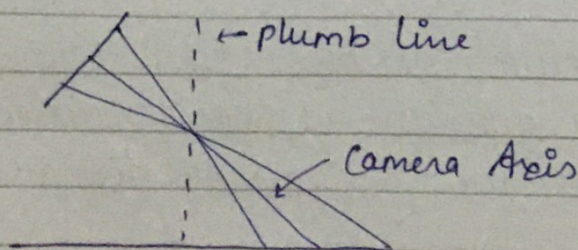
① Vertical Photograph :-



- When camera axis coincides with the line of gravity, photographs obtained are called as vertical photographs.
- In vertical photographs, photoplane is parallel to Datum plane. But it is extremely difficult to keep the axis perfectly vertical because of unavoidable tilt of the aircraft.

NOTE If camera axis is tilted slightly b/w $1^\circ - 3^\circ$ from the vertical then it is called as 'Tilted photograph'.

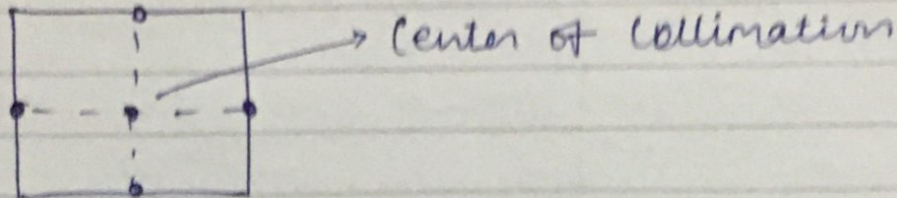
Oblique photographs :-



When camera axis is considerably inclined to the vertical then photographs obtained are called as oblique photographs.

Focus plane & Fiducial mark :-

Focus plane is the plane in an aerial camera on which all the incident rays are brought to focus.

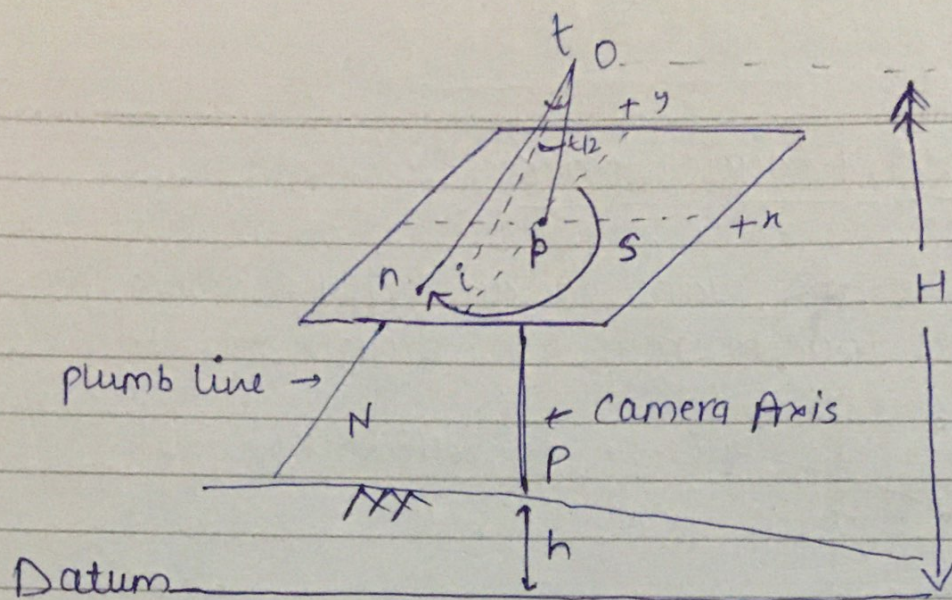


- Fiducial marks are provided in camera image plane to have a rectangular coordinate system for the measurement of various points.
- Fiducial marks are 4 or 8 in numbers.
- Lines joining opposite fiducial marks intersect at a point called as center of collimation.
- In aerial cameras center of collimation and principle point should be same point.
- principle point is a imp. reference point where camera axis intersect the photograph.

BASIC DEFINITION :-

Exposure station O :-

point in the space occupied by camera lens at the time of exposure.



Flying height :-

It is elevation of exposure station above datum.

Altitude :- $(H-h)$

It is elevation of exposure station from earth surface

Angle of tilt :- t

It is vertical angle b/w plumb line & camera axis at the time of exposure.

principle point :-

It is the point on photograph where camera axis intersect the photograph

NADIR POINT :-

It is point where plumb line intersect the photograph

Isocenter i

It is point where angle bisector of angle of tilt intersect the photograph.

$$pi = f \tan(t/2)$$

- In case of vertical photograph principle point, isocenter and Nadir point coincide with each other

ANGLE OF SWING:-

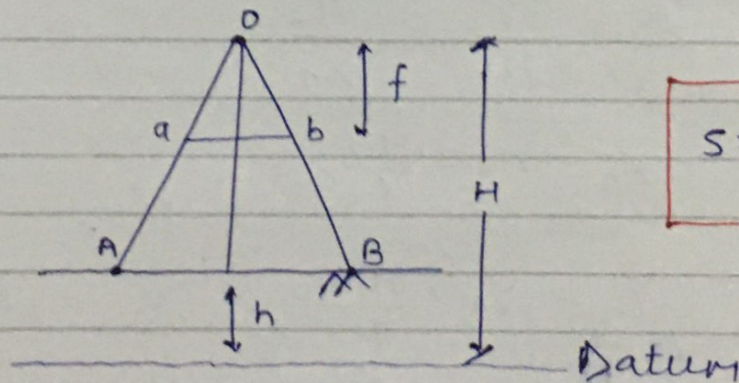
It is a clockwise Angle measured from +ve y-axis upto Nadir point.

- Angle of Rotation = Angle of swing. - 180°

SCALE OF VERTICAL PHOTOGRAPH:-

$$\text{Scale (S)} = \frac{\text{photo distance}}{\text{Ground distance}}$$

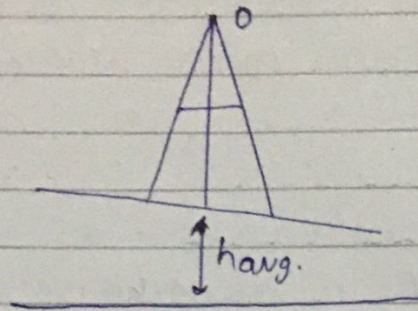
Case 1: on Flat Ground:-



$$S = \frac{ab}{AB} = \frac{f}{H-h}$$

- if scale of the photograph elevation of the ground changes then scale of the photograph also changes
- Higher the elevation of ground point above datum larger is the scale.

Case-2 on variable terrain:-



$$S_{avg.} = \frac{f}{H - h_{avg.}}$$

Case-3

DATUM SCALE:-

In this case all the points on the ground are projected vertically downward on Datum.

$$S_{Datum} = \frac{f}{H}$$

Ground coordinates :-

$$S = \frac{f}{H-h} = \frac{x_a}{X_a}$$

$$X_A = x_a \times \frac{H-h}{f}$$

$$Y_A = y_a \times \frac{H-h}{f}$$

Que. A line ~~ab~~ measures 11 cm on a photograph taken with a camera having $f = 21.5$ cm.

The same line measures 3 cm on a map drawn to a scale of 1:45000. Calculate flying height of aircraft if avg. Altitude of Area is 350 m above Datum.

$$S_p = \frac{ab}{AB} = \frac{f}{H-h}$$

$$\frac{11}{AB} = \frac{21.5}{H-350}$$

$$S_m = \frac{1}{45000} = \frac{3 \times 10^{-2}}{AB} \Rightarrow \underline{AB = 1350 \text{ m}}$$

$$\frac{11}{1350} = \frac{21.5}{H-350} \Rightarrow H = \underline{2988.636 \text{ m}}$$

Que. Determine Ground distance b/w two points A & B having elevation of 650 m. and 250 m above Datum respectively. These Ground Stations appear on a vertical photograph obtained with a camera having focal length $f = 250 \text{ mm}$ and flying height of the Aircraft is 2700 m. photo coordinates are as given below.

Pt	x cm	y cm
a	+3.65	+2.54
b	-2.25	+5.59

$$X_A = x_a \frac{H-h_a}{f} = \frac{2700-650}{25} \times 3.65 = +299.3 \text{ m}$$

$$Y_A = \frac{2700-650}{25} \times 2.54 = +208.28 \text{ m}$$

$$X_B = \frac{2700-250}{25} \times -2.25 = -220.5 \text{ m}$$

$$Y_B = \frac{2700-250}{25} \times 5.59 = \underline{547.82 \text{ m}}$$

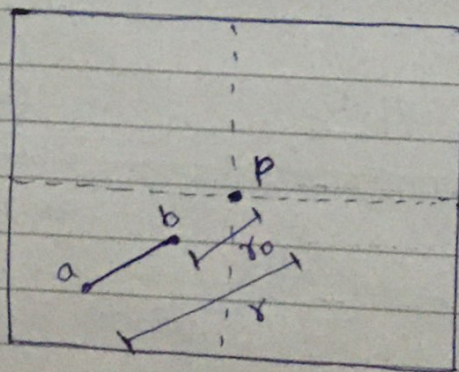
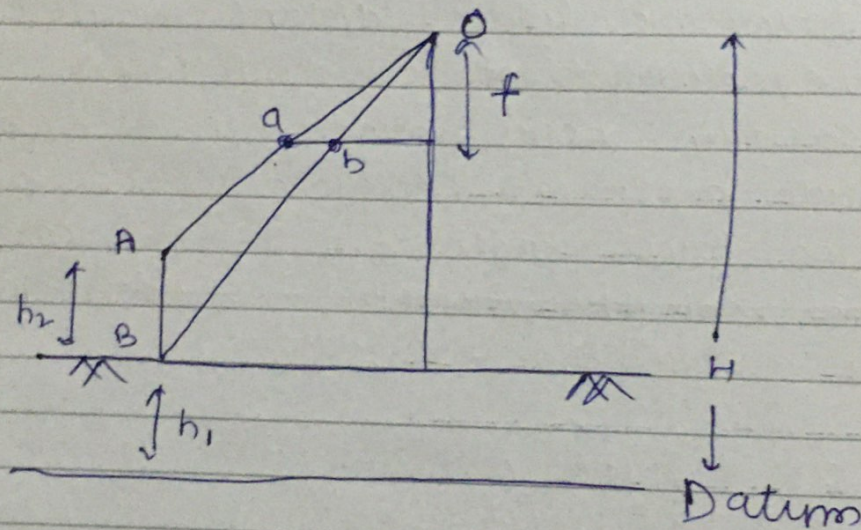
$$AB = \underline{\underline{620.86 \text{ m}}}$$

RELIEF DISPLACEMENT:-

Relief Displacement is lateral shift in photographic position of image due to its Relief.

Relief is elevation of object above or below Datum.

If the object is above Datum then Relief Displacement is radially outward and vice-versa.



$$d = r - r_0$$

$$d = \frac{r h_2}{H - h_1}$$

$r \rightarrow$ distance of top of image from principle point

Relief displacement are very useful for determining Height of various objects which appear on an arial photograph

Ex: Tower, chimney, Building.

- Relief displacement $\uparrow$, with elevation of object $\uparrow$ (h_2 from datum)
- Relief Displacement $\uparrow$, Height of object $h_2 \uparrow$
- Relief Displacement $\downarrow$, Flying Height $\uparrow$ (H)
- Relief Displacement = 0, @ principle plane.

Que: A Tower AB, 80m Height Appears in a Vertical photograph. Flying Height is 3000m. Distance of Top of the tower from the principle point on photograph is 7.25 cm, calculate Displacement of Top of the tower with respect to image of it's bottom if elevation of Bottom of the tower is from Datum = 1150 m.

$$d = \frac{7.25 \times 80}{3000 - 1150} = 0.31 \text{ cm}$$

Que: Images of A & B of the Base and Top respectively of A - 150 m High chimney observed in a vertical photograph of scale 1:10000. Determine position of a, Given that b is 70 mm from the principle point. if $f = 125 \text{ mm}$ and assume chimney at datum

$$h_1 = 0 \Rightarrow d = \frac{r h_2}{H}$$

$$d = \frac{70 \times 150}{1250} \quad \left\{ \begin{array}{l} S_d = \frac{f}{H} = \frac{125 \times 10^{-3}}{1250} \\ H = \frac{f}{S_d} = \frac{125 \times 10^{-3}}{1/10^4} \\ H = 1250 \text{ m} \end{array} \right.$$

$$d = 8.4 \text{ mm}$$

$$a = b - d = 70 - 8.4 = 61.6 \text{ mm}$$

PHOTOGRAPHIC OVERLAP:-

In Aerial survey two adjacent photographs are taken such that they have some common coverage

Common coverage along the flight line is known as longitudinal overlap, End lap, forward overlap or simply overlap, $P_L = 60\%$

Common coverage b/w two adjacent flight strip is called as side or lateral overlap

$$P_s = 30\%$$

Advantages of overlap:-

- 1) Distortion of photograph is caused due to tilt and relief displacement which is more at the edges, such portion can be discarded and only central portion can be considered
- 2) Some Gaps of uncovered area would be left out if flight lines are not straight lines or time of exposure is not proper.
- 3) Stereoscopic viewing $\rightarrow$ 3D viewing.

Number of photographs:-

METHOD-1 When total ground area to be covered is given

Number of photographs, $N = \frac{A}{a}$

$a \rightarrow$ ground area covered by one photograph

$$a = (1 - P_L) \frac{l}{s} \times (1 - P_S) \frac{w}{s}$$

$l \rightarrow$ length of photograph

$P_L \rightarrow$ longitudinal overlap

$P_S \rightarrow$ side overlap

$w \rightarrow$ width of photograph

$s \rightarrow$ scale of photograph

METHOD 2

When Ground length (L) and Ground width (W) to be covered are given therefore,

$$N = N_1 \times N_2$$

Where, $N_1 \rightarrow$ Number of photographs in the dirⁿ of flight line

$N_2 \rightarrow$ Number of flight lines.

$$N_1 = \left(\frac{\frac{L}{(1 - P_L) l/s} + 1 \right), N_2 = \left(\frac{\frac{W}{(1 - P_S) w/s} + 1 \right)$$

Time interval b/w exposure :-

$$T = \frac{\text{Ground length covered by one photograph}}{\text{speed of Aircraft}}$$

NOTE

Ground length covered by one photo = equal to distance b/w two exposure stⁿ = Air Base

INTERVALometer:- It is used to determine the time interval b/w two exposure station.

DRIFT:- Drift is lateral shifting of photographs due to cross winds & components.

CRAB:- When edges of the photograph are not parallel to the dirⁿ of flight line at the time of exposure, then crab is introduced.

NOTE To avoid Drift Crab is introduced.

Que Determine the number of photographs required to cover Area of 500 km^2 if scale is $1/10000$ and photo size size $230 \times 230 \text{ mm}$.

$$P_L = 60\% , P_S = 30\%$$

$$N = \frac{A}{a} = \frac{500 \times 10^6}{\left((1-0.6) \times \frac{0.23}{1/10000} \right) \left((1-0.3) \times \frac{0.23}{1/10000} \right)}$$
$$= 337.56$$
$$\approx 338$$

Que Determine the number of photographs required to cover an Area of $20 \text{ km} \times 25 \text{ km}$.

$$S = 1/10000 , P_L = 0.6 , P_S = 0.3 , 230 \times 230 \text{ mm}$$

$$N = N_1 \times N_2$$

$$N = \left[\frac{\frac{25 \times 10^3}{(1-0.6) \times \frac{0.23}{1/10000}} + 1 \right] \times \left[\frac{\frac{20 \times 10^3}{(1-0.3) \times \frac{0.23}{1/10000}} + 1 \right]$$

$$N = ~~28~~ 28.17 \times 13.42 = 29 \times 14 = 406$$

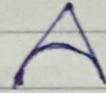
CURVES

Curves are defined as arcs with some finite radius provided b/w intersecting straight line to gradually negotiate change in dirⁿ.

This change in dirⁿ of straight line may be in horizontal or a vertical plane, which results in horizontal or vertical curve respectively.

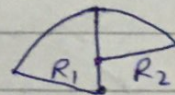
Horizontal curve:-

1) Simple Circular Curve:-



A simple circular curve consists of an arc of a circle which is tangential to the straight line.

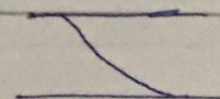
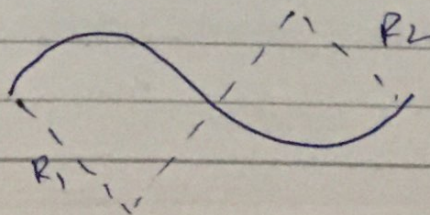
2) Compound curve:- A compound curve consists of two circular arcs of different radii with their centres of curvature on the same side of a common tangent.



3) Reverse curve / S curve / serpentine curve:-

When two simple circular curves of equal or different radii are joined together, the resultant curve is called as Reverse curve

having opposite dirⁿ of curvature



These are provided b/w two parallel lines or when angle b/w them is very small

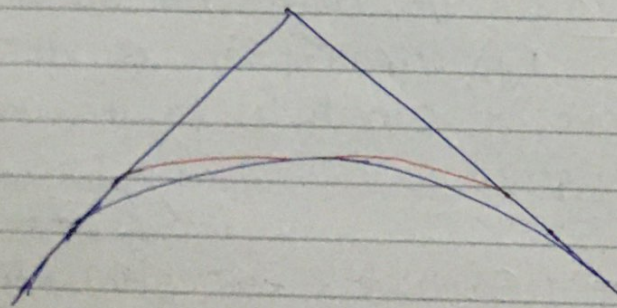
They are common in Railway but avoided in Highway

4) Transition curve / Easement curve :-

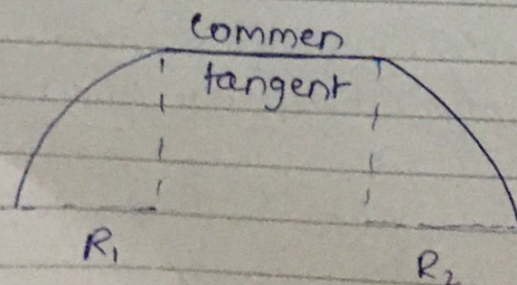
It is a curve of varying radius introduced b/w a straight line and a circular curve.

Radius of transition curve varies b/w some finite value to infinite value, or vice versa

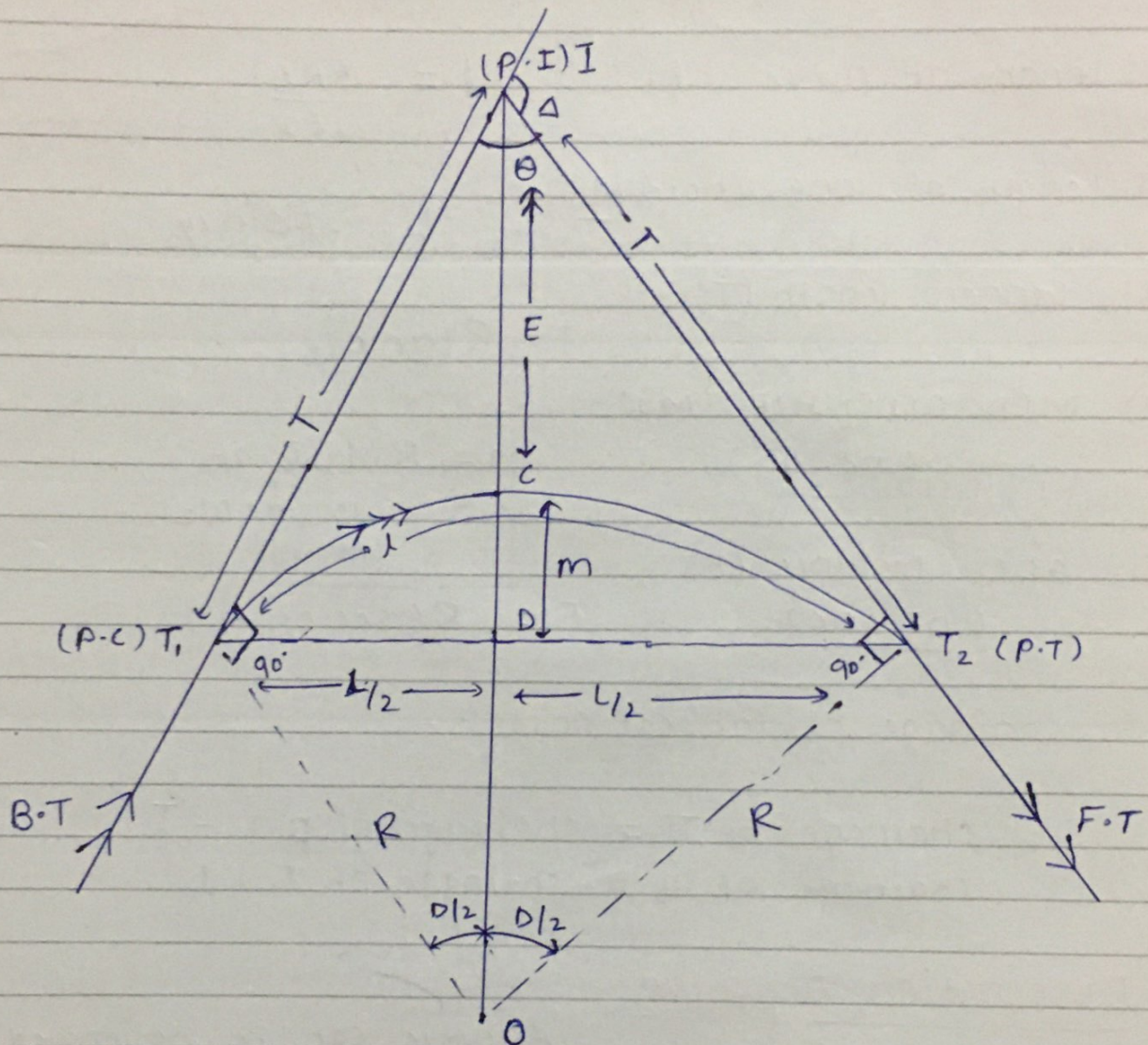
Transition curve shall also be provided b/w two branches of reverse curve and compound curve.



5) Broken Back curve :-



SIMPLE CIRCULAR CURVE:-



- | | |
|---|---|
| B.T → Back tangent | L → length of curve |
| F.T → Forward tangent | m → mid-ordinate |
| P.C → point of curve | E → apex distance / External distance |
| P.T → point of tangency | |
| P.I → point of intersection | |
| Δ → deflection angle | |
| θ → angle of intersection | |
| $\angle T_1 O T_2$ → central angle = Δ | |
| $T_1 I$ & $T_2 I$ → tangent length | |
| $T_1 T_2 = L$ → long chord | |

ELEMENTS OF SIMPLE CIRCULAR CURVE:-

1) length of curve (l):- $l = \frac{\pi R \Delta}{180^\circ}$

2) length of long chord (L):- $L = 2R \sin \Delta/2$

3) tangent length (T):- $T = R \tan \Delta/2$

4) MID ORDINATE (M):-
 $(OC - OD)$ $M = R(1 - \cos \Delta/2)$
 $M = R \text{ versine } \Delta/2$

5) Apex distance (E):-
 $(OI - OC)$ $E = R(\sec \Delta/2 - 1)$

6) chainage of tangent points :-

$$\text{chainage of } T_1 = \text{chainage of P.I.} - T$$

$$\text{chainage of } T_2 = \text{chainage of } T_1 + l$$

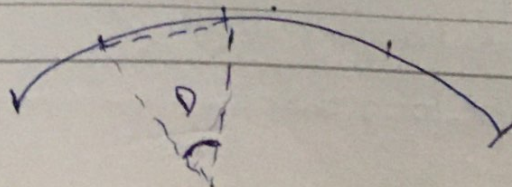
Designation of curve:-

A curve can be designated with Radius R or degree of curve

Degree of curve is the angle subtended by an arc or a chord of specified length at the center

Arc Definition:- 20m or 30m

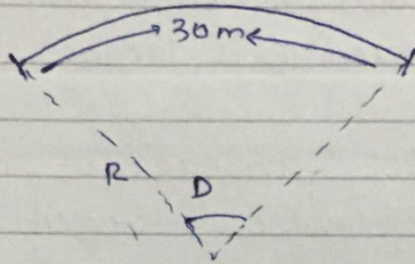
chord Definition:- 20m or 30m



RELATIONSHIP b/w Radius & Degree of curve:-

ARC DEFINITION:-

Case-I For 30 m Arc



$$D = \frac{1719}{R}$$

$$R \times D \times \frac{\pi}{180} = 30$$

$$D = \frac{30 \times 180}{\pi R}$$

$$D = 1718.87 / R$$

Case-II For 20 m Arc

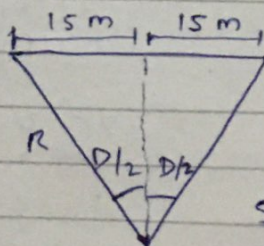
$$D = \frac{20 \times 180}{\pi R}$$

$$D = \frac{1145.91}{R}$$

$$D = \frac{1146}{R}$$

CHORD DEFINITION:-

Case-I 30 m chord:-



$$\frac{D}{2} \times \frac{\pi}{180} = \frac{15}{R}$$

$$\sin \frac{D}{2} = \frac{15}{R}$$

Since R is very large so
 $\sin \frac{D}{2} \approx \frac{D}{2}$

$$D = \frac{15 \times 2 \times 180}{\pi R}$$

$$D = \frac{1719}{R}$$

Case-2 20m chord:-

$$D = \frac{1146}{R}$$

NOTE As Degree of curve is inversely proportional to Radius. Hence, For Flat curve, D is small. Whereas, for sharp curve D is large.

Que. $R = 1000 \text{ m}$, $D = 60^\circ$, chainage of P.I. = 2000 m. Determine length of curve, tangent length, long chord, midordinate, Apex distance, chainages of T_1 & T_2 and Degree of curve for 30 m Arc.

$$l = \frac{\pi R D}{180^\circ} = 1047.198 \text{ m}$$

$$T = R \tan \frac{D}{2} = 577.35$$

$$L = 2R \sin \frac{D}{2} = 1000$$

$$M = R (1 - \cos \frac{D}{2}) = 133.97 \text{ m}$$

$$E = R (\sec \frac{D}{2} - 1) = 154.7 \text{ m}$$

$$\begin{aligned} \text{chainage of } T_1 &= \text{chainage of P.I.} - T \\ &= 2000 - 577.35 \\ &= 1422.65 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{chainage of } T_2 &= \text{chainage of } T_1 + L \\ &= 1422.65 + 1047.198 \\ &= 2469.848 \text{ m} \end{aligned}$$

$$D = \frac{1719}{R} = 1.719^\circ$$

$$O_n = GD = OG - OD$$

$$OD = R - O_0$$

$$OG = \sqrt{R^2 - n^2}$$

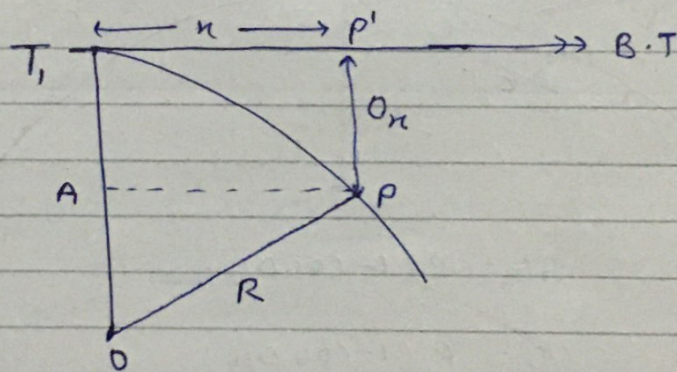
$$O_n = \sqrt{R^2 - n^2} - (R - O_0)$$

$$O_n = O_0 + (\sqrt{R^2 - n^2} - R)$$

$$O_n = O_0 + R \left[\left(1 - \frac{n^2}{R^2} \right)^{1/2} - 1 \right]$$

$$O_n = O_0 - \frac{n^2}{2R}$$

2. perpendicular offset from tangent :-

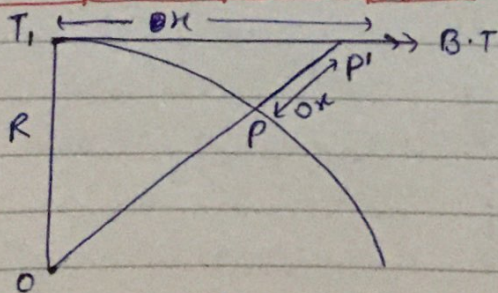


$$O_n = OT_1 - OA$$

$$O_n = R - \sqrt{R^2 - n^2}$$

$$O_n = \frac{n^2}{2R}$$

3. Radial offset from tangent :-



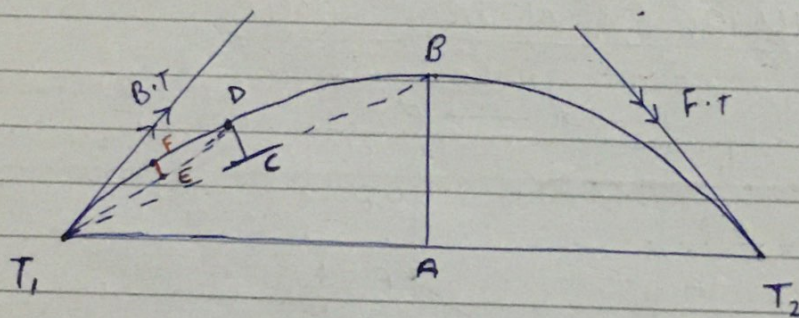
In this method info. of center of the curve is also required.

$$O_n = OP' - OP$$

$$O_n = \sqrt{R^2 + n^2} - R$$

$$O_n = \frac{n^2}{2R}$$

4. Successive Bisection of Arc :-



$$AB = R(1 - \cos D/2)$$

$$CD = R(1 - \cos D/4)$$

$$EF = R(1 - \cos D/8)$$

5. OFFSET From chord produced :-

It is Best method for setting out a long curve by linear method that is when theodolite is not available.

$$\left\{ \begin{array}{l} \frac{R}{20} < 180 \rightarrow \text{sharp} \end{array} \right.$$

SETTING OUT A SIMPLE CIRCULAR CURVE:-

Setting out a curve is a process of locating various points along the length of curve at equal and convenient distances.

Distance b/w two consecutive points on the curve is called as peg interval.

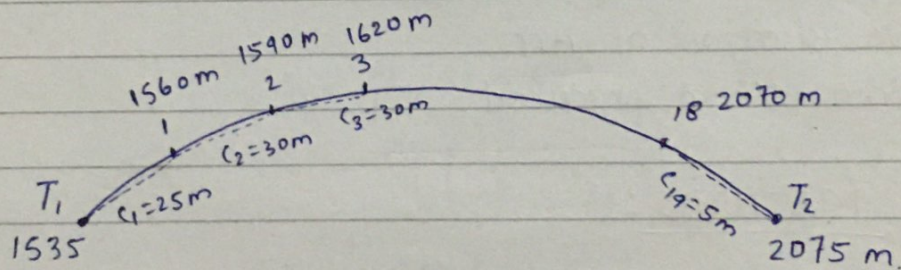
Generally peg interval is 20 m or 30 m. But for sharp curve it may be further reduced.

Let chainage of $T_1 = 1535$ m

Length of curve = 540 m.

Therefore chainage of $T_2 = 2075$ m

peg interval = 30 m.



The chainage of first point on the curve should be selected such that it becomes a full station that is multiple of chain length. Hence, chainage of first point on curve 1560 m.

Therefore $C_1 = 25$ m (1560 - 1535 m)

$$\text{Number of full chords} = \frac{540 - 25}{30} = 17.16$$

There Number of full chords = 17

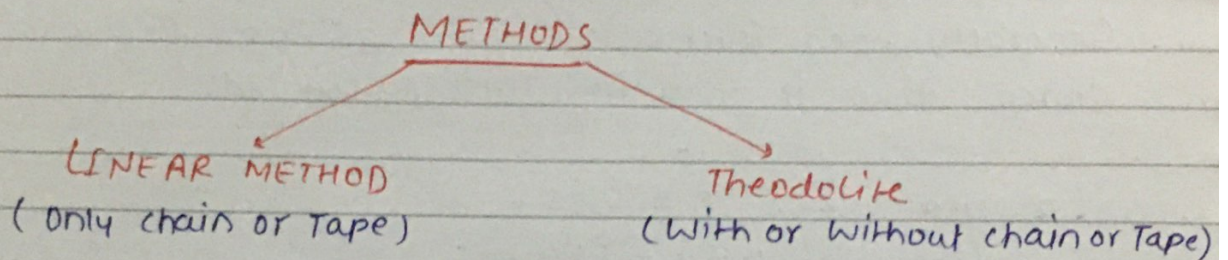
$$C_2 = C_3 = C_4 = \dots = C_{18} = 30 \text{ m.}$$

Therefore length of last sub chord = 2075 - 2070 = 5 m

Generally First and last chords are sub chord.

NOTE If chainage of $T_1 = m \text{ chain} + n \text{ link}$
then chainage of first point on curve
will be $(m+1) \text{ chain}$

METHOD OF SETTING OUT OF CIRCULAR CURVE :-

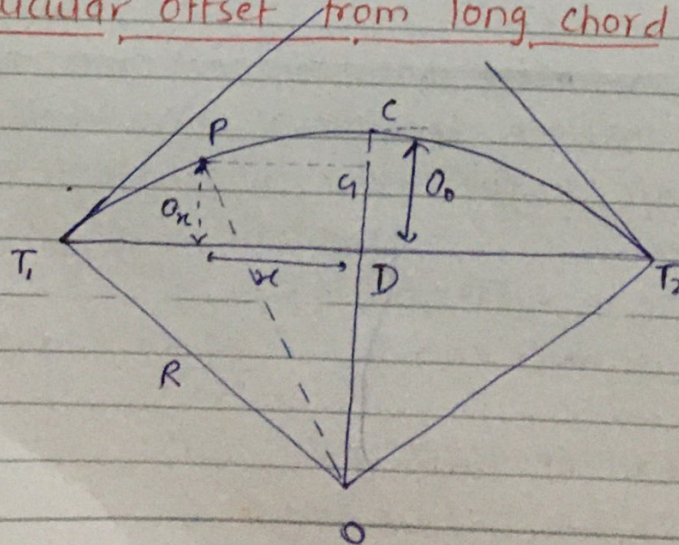


1. perpendicular offset from long chord
2. perpendicular offset from tangent
3. Radial offset from tangent
4. Successive Bisection of Arc
5. offset from chord produced

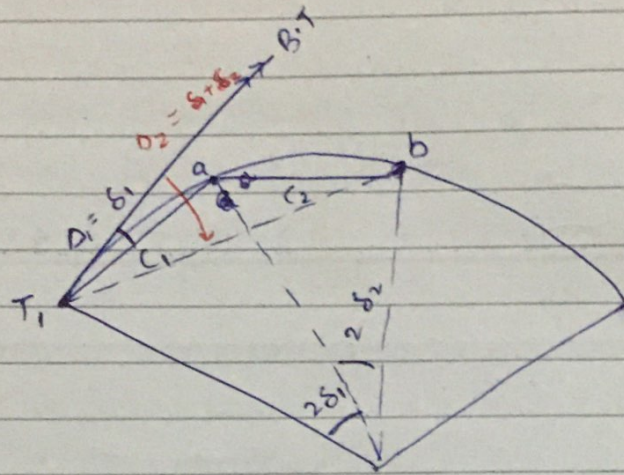
1. Rankine method
2. Two Theodolite Method
3. Tacheometric Distance Method

LINEAR METHODS :-

1. perpendicular offset from long chord :-



1. One theodolite method / Tangential angle Method / Deflection angle method / Rankine Method:-

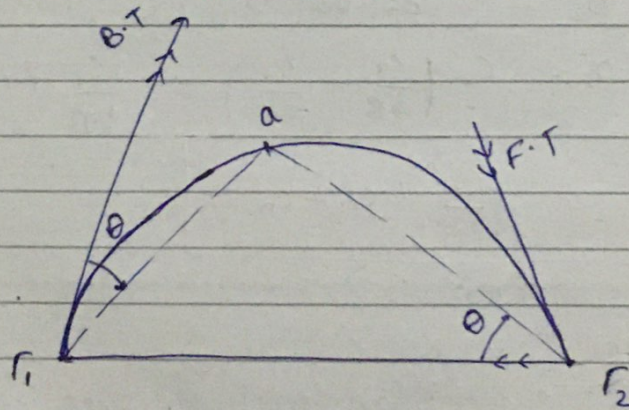


$$\delta_1 = \frac{C_1}{2R} \times \frac{180}{\pi}$$

$$\delta_1 = 1719 \frac{C_1}{R}$$

$$\delta_2 = \frac{C_2}{2R} \times \frac{180}{\pi}$$

2. Two theodolite Method:-



This method is very useful when ground is undulating, rough and not suitable for linear measurements. In this method two theodolites are used and linear measurements are completely eliminated. It is based on principle that angle subtended b/w tangent & Chord is equal to angle subtended by chord in opposite segment.

Que. Two tangents intersect at chainage 1190 m.,
Deflection angle Δ is 36° . Calculate all the
necessary data for setting out a curve of 300 m Radius.
peg interval is 30 m.

$$\text{Length of curve } L = \frac{\pi R \Delta}{180^\circ} = 188.49 \text{ m.}$$

$$\text{Tangent length } T = R \tan \frac{\Delta}{2} = 97.47 \text{ m.}$$

$$\begin{aligned} \text{chainage of } T_1 &= \text{chainage of P.I.} - T \\ &= 1190 - 97.47 = 1092.53 \text{ m.} \end{aligned}$$

$$\begin{aligned} \text{chainage of } T_2 &= \text{chainage of } T_1 + L \\ &= 1092.53 + 188.49 = 1281.02 \text{ m} \end{aligned}$$

$$C_1 = 1110 - 1092.53 = 17.47 \text{ m}$$

$$\begin{aligned} \text{Number of full chords} &= \frac{188.49 - 17.47}{30} \\ &= 5.7 = 5 \text{ Numbers.} \end{aligned}$$

$$C_2 = C_3 = C_4 = C_5 = C_6 = 30 \text{ m.}$$

$$C_7 = 1281.02 - 1260 = 21.02 \text{ m.}$$

$$\delta_1 = \frac{C_1}{2R} \times \frac{180^\circ}{\pi} = 1^\circ 40' 5.74''$$

$$\delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \frac{C}{2R} \times \frac{180^\circ}{\pi} = 2^\circ 51' 53.24''$$

$$\delta_7 = \frac{C_7}{2R} \times \frac{180^\circ}{\pi} = 2^\circ 0' 26.14''$$

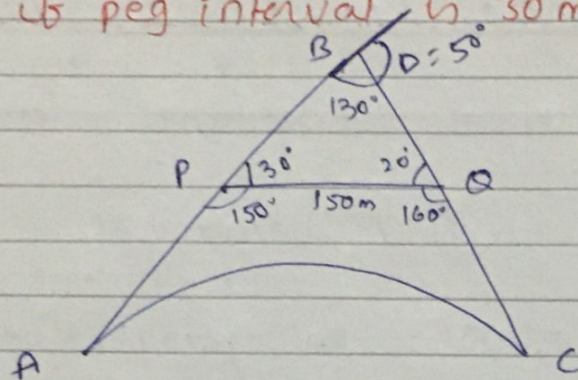
Que. Two straight line AB & BC meet at an inaccessible
point B which are to be connected by a simple
circular curve of $R = 600 \text{ m}$. Two points P & Q were
selected on AB & BC respectively and following
Data was obtained

$$\angle APQ = 150^\circ$$

$$\angle CQP = 160^\circ$$

$$PQ = 150 \text{ m.}$$

Determine chainage of B if chainage of P is 1600 m. and also determine, deflection angle Δ , if peg interval is 30 m.



$$\text{Ch. of B} = \text{ch. of P} + PB$$

$$\frac{PB}{\sin 20^\circ} = \frac{150}{\sin 130^\circ}$$

$$PB = 66.97 \text{ m}$$

$$\text{Ch. of B} = 1600 + 66.97 = 1666.97 \text{ m.}$$

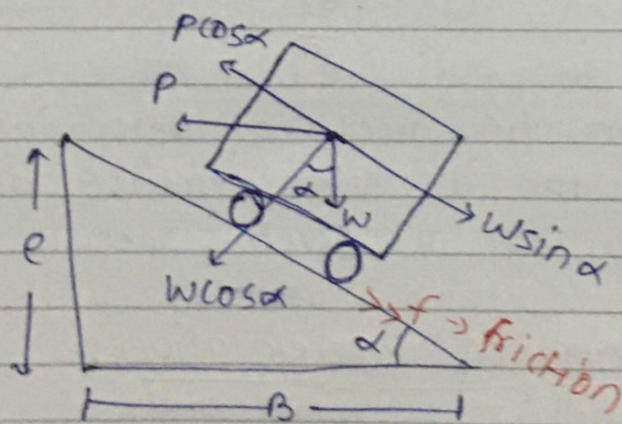
Transition curve :-

- It is a curve of varying Radius introduced b/w a straight line & circular curve
- It provides a gradual change from straight line to a circular curve and again from circular curve to a straight line.

BASIC criteria for design transition curve :-

- It should be tangential to straight line and also meet circular curve tangentially at the junction.
- It's curvature should be Zero ($r = \infty$) at one end where it meets the straight line, and it should be equal to the radius of circular curve at its junction with circular curve.
- Rate of increase of curvature along the transition curve should be equal to rate of increase of superelevation.
Therefore length of transition curve should be such that full superelevation is achieved at the junction with circular curve.

SUPERELEVATION:-



$$P \cos \alpha = W \sin \alpha$$

$$\tan \alpha = P/W$$

$$\frac{e}{B} = \frac{P}{W} = \frac{mv^2/r}{mg} \Rightarrow$$

$$e = \frac{Bv^2}{gr}$$

Max. Limit of $e = 7\%$

It is the vertical distance by which outer end of the ~~road~~ road is raised above the inner one

The value of superelevation can not be as high as possible, Because high superelevation can cause toppling of the vehicle

In such a case Either large radius is used or velocity is reduced

MAX. CENTRIFUGAL RATIO :-

$$C.R = \frac{P}{W} = \frac{V^2}{gr}$$

To avoid inconvenience to passenger max. value of centrifugal ratio is restricted to

For Highway $\rightarrow 1/4 \Rightarrow V^2/gr = 1/4$

for railway $\rightarrow 1/8$

$$\Downarrow \\ V = \sqrt{\frac{gr}{8}}$$

$$V = \frac{\sqrt{gr}}{\sqrt{4}}$$

above eqⁿ can be used to determine Max. speed for a given radius or a min. radius for a given speed.

Effect of Friction on Superelevation :-

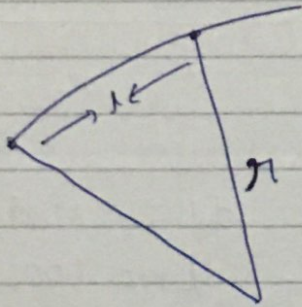
frictional force in lateral dirⁿ b/w tire and road reduces ~~effect~~ of superelevation Value

For safe design

$$\mu + \tan \alpha \geq \frac{v^2}{gr}$$

DERIVATION OF Transition curve Eqⁿ:-

a curve of variable radius of required length is inserted b/w straight Road and a circular curve such that centrifugal force increases uniformly and gradually along the length of transition curve



$$P \propto 1$$

$$P = \frac{mv^2}{r}$$

m & v are constant

$$P \propto \frac{1}{r}$$

$$L \propto \frac{1}{r}$$

~~is~~

$$Lr = \text{constant}$$

clothoid Eqⁿ

Euler spiral Eqⁿ

Glover spiral Eqⁿ

Ideal transition curve Eqⁿ

LENGTH OF TRANSITION curve:-

① such that e is applied at a uniform rate along the curve.

Let $1/n$ is the rate @ which ^{superelevation} is provided and e is total super-elevation to be provided then length of curve will be

$$L = e / \frac{1}{n}$$

$$L = ne$$

②

Such that rate of change of radial acceleration is within desired limit

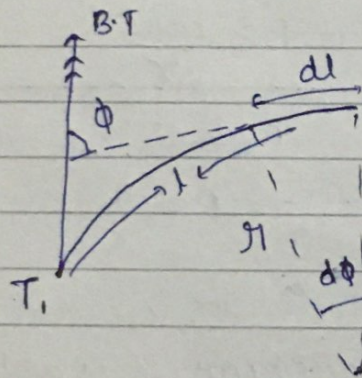
$$\alpha = \frac{V^2/r - 0}{t}$$

$$\alpha = \frac{V^2/r}{L/V}$$

$$L = \frac{V^3}{\alpha r}$$

$$\alpha \rightarrow (m/s^2)/s$$

INTRINSIC EQⁿ OF IDEAL TRANSITION CURVE:



$$dl = r d\phi$$

$$Lr = \text{const.} = K$$

$$r = K/L$$

$$\int d\phi = \int \frac{L}{K} dl$$

$$\phi = \frac{l^2}{2K} + C$$

$$l \rightarrow 0, \phi \rightarrow 0, C = 0$$

$$\phi = \frac{l^2}{2K}$$

NOTE at the end of transition curve $l = L, r = R$
therefore $LR = K$ (end point)

$$\phi = \frac{l^2}{2LR}$$

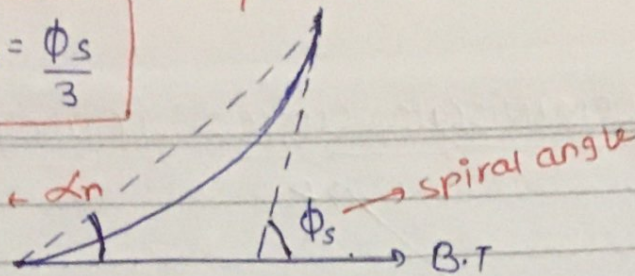
valid at every point

*

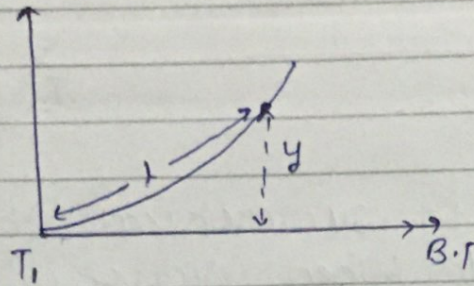
only valid for ideal transition curve!

$$\alpha_n = \frac{\phi_s}{3}$$

polar deflection angle

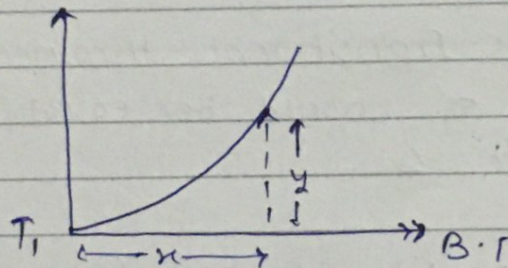


CUBIC SPIRAL CURVE :-



$$y = \frac{l^3}{6RL}$$

CUBIC PARABOLA / Fraud Transition curve :-



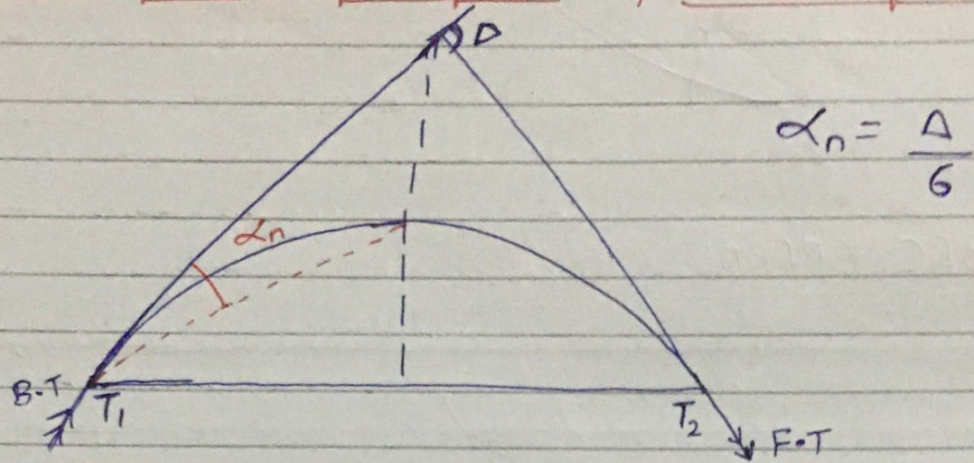
$$y = \frac{x^3}{6RL}$$

Cubic spiral more closely resembles ideal transition curve in comparison to cubic parabola

Setting of cubic parabola is more easy than cubic spiral therefore cubic parabola is commonly used.

AFTER the invention of electronic equipments any curve can be set out Hence, cubic spiral is used these days and cubic parabola is obsolete.

BERNOULLI'S LEMNISCATE curve:- / Autogeneous curve



This curve is symmetrical therefore it can be used when curve is transitional throughout without a circular curve in b/w them.

When curve is Transitional throughout polar deflection angle α should be equal to $\frac{1}{6}$ th of deflection angle Δ .

Shape of Bernoulli's Lemniscate curve is similar to Actual path traced by the vehicle when turning freely on a curve, therefore it is also called as Autogeneous curve.

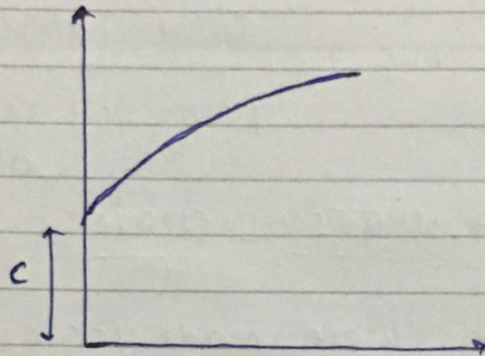
VERTICAL CURVE:-

A vertical curve is used to correct two different gradient lines of Highway and Railway.

Vertical curve can be parabolic or circular.

parabolic curve is preferred over circular curve because of following reason:

- 1) It is flatter at top and provides longer sight distance
- 2) It is simple to layout
- 3) Rate of change of gradient is constant



$$y = ax^2 + bx + c$$

$$\frac{dy}{dx} = 2ax + b$$

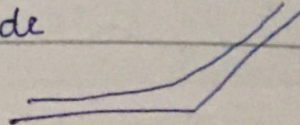
$$\frac{d^2y}{dx^2} = 2a$$

Types of vertical curve :-

vertical curve

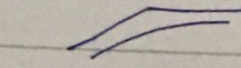
Sag curve / valley curve

- 1) Down gradient followed by up gradient
- 2) Steep down gradient followed by mild down grade
- 3) Mild upgrade followed by steep upgrade



Summit curve

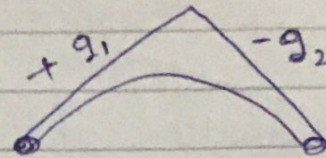
- 1) Upgrade followed by downgrade
- 2) Steep upgrade followed by mild up grade
- 3) Mild downgrade followed by steep downgrade



3) Mild downgrade followed by steep downgrade

Total change of Grade:-

$$g_1 - (-g_2)$$



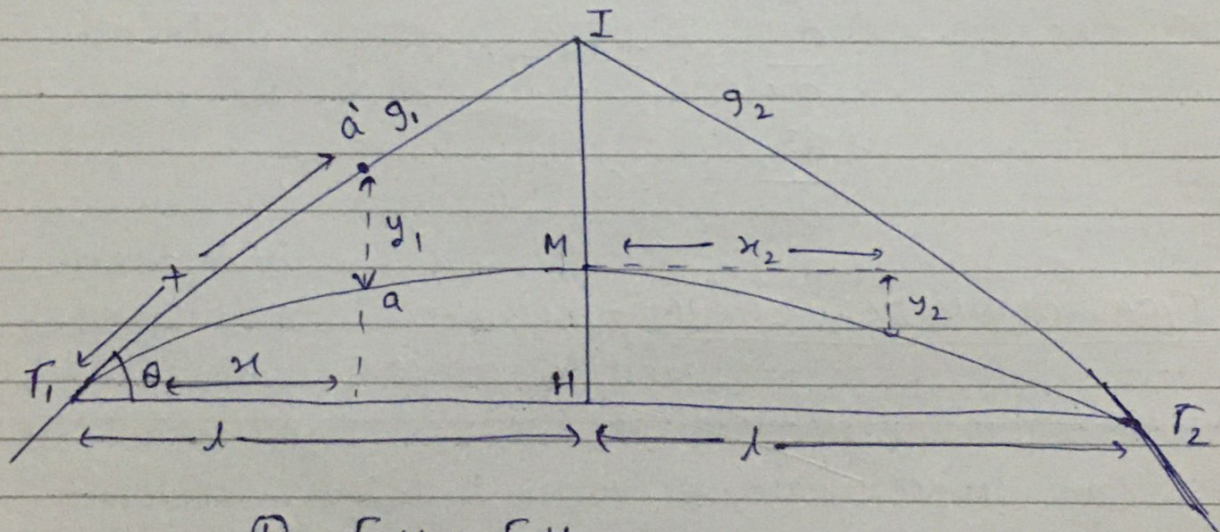
It is algebraic difference of 2 gradients

LENGTH OF VERTICAL CURVE:-

length of vertical curve = $\frac{\text{Total change of grade}}{\text{permissible rate of change of grade}}$

ASSUMPTIONS OF A VERTICAL CURVE:-

Following assumptions were made for a verticle curve used in practice which are generally Flat curve that is these values are valid for g_1 & $g_2 \leq 4\%$



- ① $T_1 H = T_2 H$
- ② $I M = M H$
- ③ $T_1 I = T_1 M = T_1 H$
- ④ $y_1 \propto x^2$

$$x \cos \theta = x$$

$$x = x \sec \theta$$

$$y_1 \propto (x \sec \theta)^2 ; \sec^2 \theta \text{ is constant}$$

$$\boxed{y_1 \propto x^2}$$

$$(5) \quad y_2 \propto x_2^2$$

SETTING OUT OF VERTICAL CURVE:-

$$\text{R.L of } T_1 = \text{R.L of } I - g_1 l$$

$$\text{R.L of } T_2 = \text{R.L of } I + g_2 l$$

$$\text{R.L of } H = \frac{\text{R.L of } T_1 + \text{R.L of } T_2}{2}$$

$$IH = \text{R.L of } I - \text{R.L of } H$$

$$IM = MH = IH/2$$

$$\text{R.L of } a' = \text{R.L of } T_1 + g_1 x$$

$$\text{R.L of } a = \text{R.L of } a' - y_1$$

$$y_1 \propto x^2$$

$$\frac{y_1}{IM} = \frac{x^2}{l^2}$$

THEORY OF ERROR

Measurement of angles and distances are done in various surveying operations, irrespective of all the precaution and precision exercise, there is all possibility for errors to creep in.

As far as possible errors should be minimized while taking the measurements and errors will still creep in their effect should be corrected or eliminated.

Types of Error :-

① Gross Error :- These are the discrepancies caused due to carelessness, misunderstanding and poor judgement.

- By careful work and adopting standard procedures mistakes can be avoided.

② Systematic Error / Cumulative Error :-

These errors follow a definite mathematical or physical law.

③ Random Error / Accidental Error / Compensating Error :-

Random errors are the errors which are left out in the measurement if mistake and systematic error are eliminated.

It is observed that accidental error tends to cancel out each other when large number of observations are made because these errors have equal chance of getting +ve & -ve.

Hence, these are also called as compensating error!
But this compensating affect is only partial.

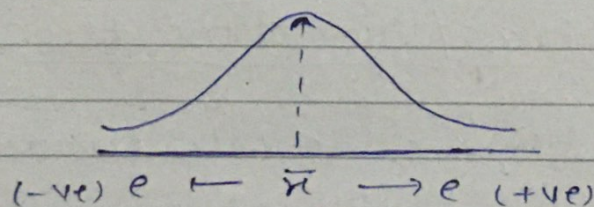
Hence, Sum of accidental error is always left out.

They obey law of chance. Hence, they can be analysed by theory of Probability

properties of Random ERROR :-

- 1) large error rarely occur
- 2) small errors are more frequent
- 3) They have equal chance of getting +ve & -ve

When a curve is plotted based on above properties a Normal distribution curve is obtained



Law of Accidental error :-

We know that

$$E = M.V - T.V$$

But T.V of a quantity is never known to us. Hence, true error is never known to us.

Instead of true value a value called most probable value (M.P.V) is commonly used.

Most probable Value of a quantity is that value which is close to true value than any other value.

Therefore, $E = M \cdot V - M \cdot p \cdot V$

Above difference is also called as Most probable Error, Residual Error, Variation.

M.p.v of a quantity can be find out if that quantity is Measured Number of times.

If all the observations are made with equal weight then most probable value is arithmetic mean of that observation.

If observations are made with different weight then $M \cdot p \cdot V = \text{Weighted Mean}$.

precision
INDICES OF PRECISION for observation of Equal weight:-

1) Standard Deviation (σ):-

$$\sigma = \pm \sqrt{\frac{\sum v^2}{n-1}}$$

$$v = M \cdot V - M \cdot p \cdot V$$

2) Variance

$$V = \sigma^2$$

3) Standard error of single observation:-

It is also called as standard error, probable error, and standard deviation.

$$\sigma_1 = \pm \sqrt{\frac{\sum v^2}{n-1}}$$

4) Standard deviation of Mean :-

$$\sigma_m = \pm \frac{\sigma}{\sqrt{n}}$$

5) Most probable error :- (e)

$$e = \pm 0.6745 \sigma$$

It is the error within which the chances of occurrence is 50% and beyond it 50%.

6) Most probable error of Mean :- (e_m)

$$e_m = \pm \frac{e}{\sqrt{n}}$$

7) Maximum Error :- (E_{max})

$$E_{max} = \pm 3.29 \sigma$$

If error is greater than E_{max} then it is treated as mistake

Que. In a survey the following 4 measurements are made by a group using same instrument
75°42'10" , 75°41'50" , 75°41'40" , 75°42'0"

Determine standard deviation, standard deviation of Mean, most probable error, most probable error of Mean, max. error and variance.

$$\sigma = \pm \sqrt{\frac{\sum v^2}{n-1}} = 12.01''$$

$$\sigma_m = \pm \frac{\sigma}{\sqrt{n}} = \pm 6.45''$$

$$e = \pm 0.6745 \sigma = \pm 8.11''$$

$$e_m = \pm \frac{e}{\sqrt{n}} = \pm 4.35''$$

$$e_{\max.} = \pm 3.29 \sigma = \pm 42.47''$$

$$V = \sigma^2 = 166.66 \text{ sec}^2$$

Weight:-

Weight of a quantity is a measure of relative trustworthiness of group of observations

Greater the precision greater will be the weight
Weight are expressed in Numbers

For example. An observation having weight = 6 is
3 times more precise than observation
having weight = 2

LAW OF ALLOCATION OF Weight:-

$$\textcircled{1} \quad W \propto \frac{1}{V} \propto \frac{1}{(S.E.)^2}$$

② Weight of a quantity measured in similar conditions are assigned in direct proportion to the number of times the quantity is measured.

③ Weight is inversely proportional to length of line

④ Sometimes weight are also allocated by personal judgement depending on prevailing field conditions. That is lower weight is allocated to the measurement done in adverse condition in comparison to measurement done in favorable conditions.

LAWS OF Weight :-

(i) Weight of arithmetic mean of Number of observation of unit wt. is equal to Number of observation.

$$\begin{array}{ll} \angle A = 40^\circ 10' & w=1 \\ \angle A = 40^\circ 20' & w=1 \end{array} \quad \text{M.P.V} = 40^\circ 15', w=2$$

(2) Weight of weighted arithmetic mean is equal to sum of individual weight

$$\begin{array}{ll} \angle A = 40^\circ 10' & w=2 \\ \angle A = 40^\circ 20' & w=3 \end{array} \quad \text{M.P.V} = 40^\circ + \frac{2 \times 10' + 3 \times 20'}{5} = 40^\circ 16' \quad \underline{w=5}$$

(3) Weight of algebraic sum of two or more quantities is equal to reciprocal of the sum of reciprocal of their individual weight

$$\begin{array}{ll} \angle A = 100^\circ & w=2 \\ \angle B = 60^\circ & w=3 \end{array} \quad A+B = 160^\circ \quad w = \frac{1}{\frac{1}{2} + \frac{1}{3}} = \frac{6}{5}$$

$$A+B=40^\circ, \quad W = \frac{1}{\frac{1}{2} + \frac{1}{3}} = \frac{6}{5}$$

- 4) If a quantity of given weight is multiplied by a factor then weight of resulting quantity is obtained by dividing the given weight by square of that factor.

$$A=60^\circ, \quad W=3$$

$$2A=120^\circ, \quad W = \frac{3}{(2)^2} = \frac{3}{4}$$

- 5) If a quantity of given weight is divided by a factor, weight of result is obtained by multiplying the given wt. by square of that factor

$$A=60^\circ, \quad W=3$$

$$A/2=30^\circ, \quad W = 3 \times 2^2 = 12$$

- 6) If a quantity is multiplied by it's own weight, weight of resulting eqⁿ is equal to reciprocal of it's own weight,

- 7) Weight of an eqⁿ remains unchanged if all the sign of the eqⁿ are reversed.

- 8) Weight of an eqⁿ remains unchanged if that eqⁿ is added to or subtracted from a constant.

$$A+B=70^\circ, \quad W=2$$

$$180^\circ + A+B = 250^\circ, \quad W=2$$

$$180^\circ - (A+B) = 110^\circ, \quad W=2$$

LAW OF propagation of Error:-

$$\text{Let } A = f(x, y, z)$$

- $\pm e_x, \pm e_y, \pm e_z$ are probable error in x, y & z respectively.

then probable error in A will be

$$e_A = \left[\left(\frac{\partial f}{\partial x} \times e_x \right)^2 + \left(\frac{\partial f}{\partial y} \times e_y \right)^2 + \left(\frac{\partial f}{\partial z} \times e_z \right)^2 \right]^{1/2}$$

Case-I

$$A = x + y + z$$

$$e_A = \pm \sqrt{e_x^2 + e_y^2 + e_z^2}$$

Case-II

$$A = xyz$$

$$e_A = \pm \left[(yz e_x)^2 + (xz e_y)^2 + (xy e_z)^2 \right]^{1/2}$$

Case-III

$$A = Kx^n, \text{ where } K \text{ is a constant}$$

$$e_A = \pm Knx^{n-1} e_x$$

Case-IV

$$A = x \pm K$$

$$e_A = \pm e_x$$

Que Determine MPV of $A+B$ if MPV of A & B are as given below

$$A = 40^\circ \pm 10''$$

$$B = 50^\circ \pm 15''$$

$$A+B = 90^\circ, \quad e_{A+B} = \sqrt{10^2 + 15^2} = \pm 18.03''$$

$$\text{M.P.V} = 90^\circ \pm 18.03''$$

Que Determine mpv of an area of rectangular if its sides are as given below,

$$a = 10\text{m} \pm 5\text{cm}$$

$$b = 20\text{m} \pm 10\text{cm}$$

$$A = a \times b = 200\text{m}^2$$

$$e_A = \sqrt{(10 \times 0.1)^2 + (20 \times 0.05)^2}$$

$$e_A = \pm 1.414\text{m}^2$$

$$\text{M.P} = 200 \pm 1.414\text{m}^2$$

Que Determine error in Area of a circle if it's radius is $r = 12 \pm 0.4\text{m}$

$$A = \pi r^2 = 452.39\text{m}^2$$

$$e_A = \pm 2\pi r e_r = \pm 30.16\text{m}^2$$

$$\text{M.P.V} = 452.39 \pm 30.16\text{m}^2$$

Que The height of a transit stⁿ has been determined from the measurement of a slope distance $L = 279.1 \pm 0.06\text{m}$ and a vertical angle $\alpha = 3^\circ 20' \pm 30''$, What is probable error in elevation of Transit stⁿ.

$$e_H = \left[\left(\frac{\partial H}{\partial L} \times e_L \right)^2 + \left(\frac{\partial H}{\partial \alpha} \times e_\alpha \right)^2 \right]^{1/2}$$

$$e_H = \left[(L \sin \alpha \times e_L)^2 + (L \cos \alpha \times e_\alpha)^2 \right]^{1/2}$$

$$e_H = \left[(\sin 3^\circ 20' \times 0.06)^2 + \left(\frac{279.1 \cos 3^\circ 20' \times 30''}{206265} \right)^2 \right]^{1/2}$$

$$e_H = \pm 0.0406\text{m}$$

Correction to Field Measurement :-

It is Mandatory to check the measurements made in the field for closing error if any the closing error is distributed to various observed quantities as per following rules.

- 1) Correction $\propto \frac{1}{\text{Weight}}$
- 2) Correction $\propto (\text{probable error})^2$
- 3) Correction $\propto \text{length of the Line}$

Ques On stn O 3 horizontal angles were Measured by closing the horizon as given below

$$\angle A = 34^\circ 10' 20'' \pm 3''$$

$$\angle B = 176^\circ 40' 32'' \pm 4''$$

$$\angle C = 149^\circ 9' 4'' \pm 5''$$

Determine correct angles.

$$A + B + C = 359^\circ 59' 56''$$

$$\text{Theoretical sum} = 360^\circ$$

$$\text{correction} = +4''$$

Let C_1, C_2 & C_3 are correction in angle A, B & C respectively

$$\text{Therefore } C_1 + C_2 + C_3 = 4''$$

We know that Correction $\propto (\text{standard error})^2$

$$C_1 : C_2 : C_3 = 3^2 : 4^2 : 5^2$$

$$C_2 = \frac{16}{9} C_1, \quad C_3 = \frac{25}{9} C_1$$

$$C_1 + \frac{16}{9} C_1 + \frac{25}{9} C_1 = 4'' \Rightarrow C_1 = 0.72''$$
$$C_2 = 1.28''$$
$$C_3 = 2''$$

Corrected angles .

$$\angle A = 34^{\circ} 10' 20.72''$$

$$\angle B = 176^{\circ} 40' 33.28''$$

$$\angle C = 149^{\circ} 9' 6''$$

THEORY OF LEAST SQUARE METHOD:-

According to this theory if measurements are of equal weight, most probable value is that value which makes sum of square of residual to a minimum value.

$$\sum v^2 = \text{Min.}$$

If measurements are of unequal weight then MPV is that value which makes sum of the product of weight and square of residual a minimum value.

$$\sum (w v^2) = \text{Min.}$$

Least square adjustment can be done in two ways

1) Normal Eqⁿ Method :-

2) Method of correlates / condition Eqⁿ Method :-

① Normal Eqⁿ Method :-

RULE 1. For observation of equal wt.

To form a normal eqⁿ for each of the unknown quantities, multiply each observation eqⁿ by the algebraic coefficient of that quantity in that eqⁿ and add the result

Rule 2. For observation of unequal wt.

To form the normal eqⁿ for each of the unknown quantity, Multiply each observation eqⁿ with the product of that unknown quantity in that eqⁿ and wt. of that observation, add the result.

Que. Find MPV of the angle A from following observation eqⁿ $A = 40^\circ 20' 15''$

$$2A = 80^\circ 40' 50''$$

$$4A = 161^\circ 21' 20''$$

$$1 \times A = 40^\circ 20' 15''$$

$$2 \times 2A = 161^\circ 21' 40''$$

$$4 \times 4A = 645^\circ 25' 20''$$

$$\underline{21A = 847^\circ 7' 15''}$$

$$A = 40^\circ 20' 20.71''$$

Que. Find the MPV of A & B from following observation eqⁿ.

given $\left\{ \begin{array}{l} A = 40^\circ 15' 21.4'' \\ B = 45^\circ 12' 18.4'' \\ A+B = 85^\circ 27' 45.2'' \end{array} \right.$

Normal eqⁿ for A

$$1 \times A = 40^\circ 15' 21.4''$$

$$1 \times (A+B) = 85^\circ 27' 45.2''$$

$$2A+B = 125^\circ 43' 6.6'' \quad \text{--- ①}$$

Normal eqⁿ for B.

$$1 \times B = 45^\circ 12' 18.4''$$

$$1 \times (A+B) = 85^\circ 27' 45.2''$$

$$A+2B = 130^\circ 40' 3.6'' \quad \text{--- ②}$$

$$A = 40^{\circ} 15' 23.2''$$

$$B = 45^{\circ} 12' 20.2''$$

Que · Determine M.P.V of angle A from following observations

$$A = 42^{\circ} 14' 20'' \quad w = 2$$

$$2A = 85^{\circ} 5' 40'' \quad w = 3$$

$$(1 \times 2) A = 84^{\circ} 28' 40''$$

$$(2 \times 3) 2A = 510^{\circ} 34' 0''$$

$$14A = 595^{\circ} 2' 40''$$

$$A = 42^{\circ} 30' 11.43'' \quad \underline{\quad}$$

PLANE TABLE SURVEYING

plane table surveying is a graphical method of surveying in which field work and plotting are done simultaneously.

It is mainly used for small and medium scale mapping ($1:10000 - 1:250000$) for topographical survey in an open area.

Before starting plane table survey at first control stations are established to cover the entire area and thereafter a suitable scale is selected.

After that surveyer starts collecting detail one by one from one of the traverse station towards all the other traverse station.

Elevations of points of observations are collected with the help of levelling, Indian clinometer or Telescopic alidade.

- Finished maps are called as topographic maps and this process is called as Cartography.

Advantages of plane table surveying:

- 1) Sighting and plotting can be done simultaneously. Hence, there is no chance of missing any detail.
- 2) No great skill is required.
- 3) Irregular objects can be plotted accurately.

4) less costly than theodolite survey.

5) It is very useful in the areas where compass survey is not useful because area is affected by local attraction.

Disadvantages:-

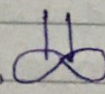
1) It can not be done in rainy season or thick dense forest

2) plane table is heavy and awkward to carry, accessories are also liable to be lost

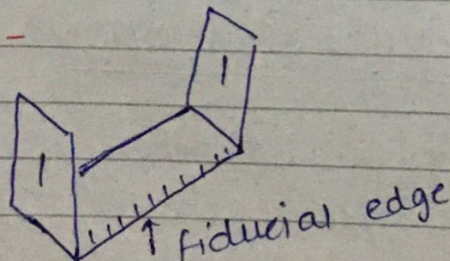
3) It becomes difficult if plan or map is to be plotted on some different scale due to absence of measurement and field notes.

Accessories used in plane table surveying:-

1) Plane table:- It is a drawing board, mounted on a tripod. With the help of wing Nut it can be clamped in any position

wing Nut 

2) Alidade:-



Alidade is a straight edge ruler used for sighting objects & drawing lines.

Line of sight should be parallel to fiducial edge

3) Level Tube :- It is used to make drawing board horizontal

4) Trough Compass :-

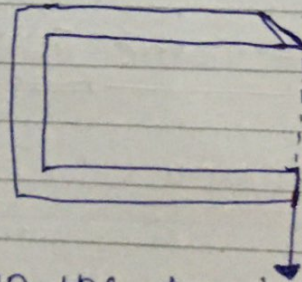


Trough compass is used to orient a plane table with respect to magnetic meridian

5) plumbing Fork :-

plumbing fork is a U shape metal frame, point of upper arm and plumb line are in same vertical line

plumbing fork is used for following purpose



- 1) Transferring ground point on the drawing sheet
- 2) centering the plane table over the station occupied by it. Which is already plotted on the drawing sheet.

NOTE :-

plumbing fork is mainly used for large scale mapping, it is not required for small scale mapping

SETTING UP the plane Table :-

- 1) Centering
- 2) levelling
- 3) orientation :-

This is the process of aligning plane table by rotating it in the horizontal plane such that plotted line becomes parallel to ground line.

If orientation of plane table is different at successive st^{ns}, relative position of plotted point will not remain same as relative position of ground points.

As the plane table is rotated during orientation, centring of plotted stⁿ is disturbed. Hence orientation & centring are complementary to each other.

METHODS OF plane Table orientation :-

- 1) Trough Compass :-

We'll place the trough compass on the top right hand side corner and draw ~~the~~ a line in N-S dirⁿ.

thereafter this line in N-S dirⁿ is made to coincide with magnetic Niddle at all the successive station.

2) by back sighting :-

On this method plane table is oriented by back sighting on previous st^n .

Most important

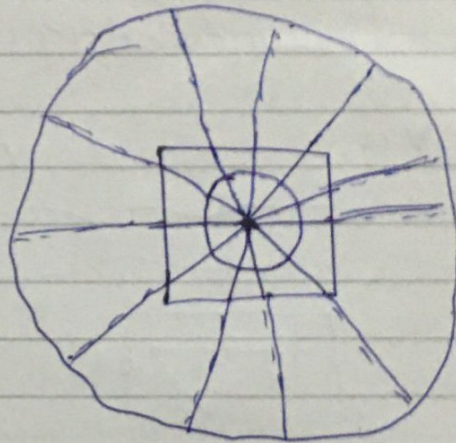
* 3) Resection :-

Methods of plane table surveying :-

1) Radiation :-

In this method instrument is set up at a st^n and rays are drawn to various st^n which

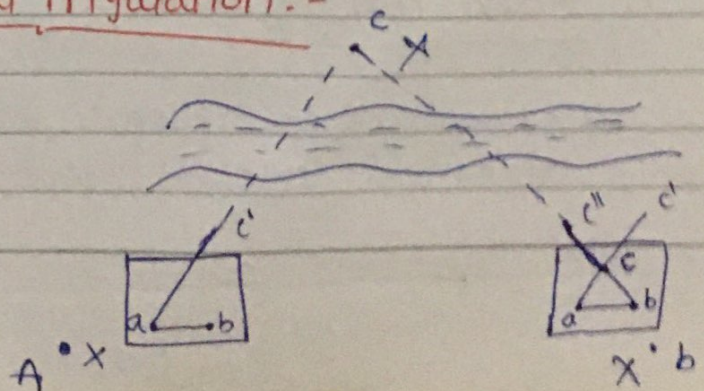
are to be plotted then distances are cut on a suitable scale after measurement



It is suitable for small Area, st^n 's are clearly visible from Instrument st^n and stations are 'accessible'

2) Traversing :- This method is same as compass or theodolite traverse in which details are collected from various Instrument st^n .

3) Intersection / Graphical Trignuation :-

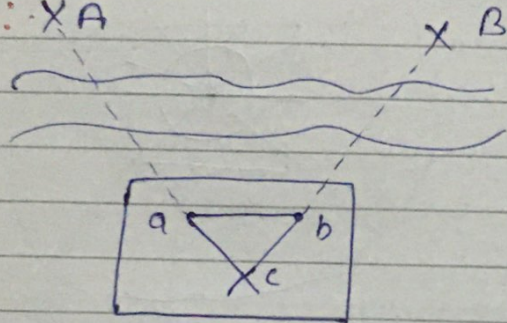


This is a method of locating a point on the drawing sheet by intersection of two rays drawn from two different st^n

• It is preferred when

- 1) Distance b/w st^n is large
- 2) Stations are inaccessible
- 3) Ground is undulating

Resection: XA

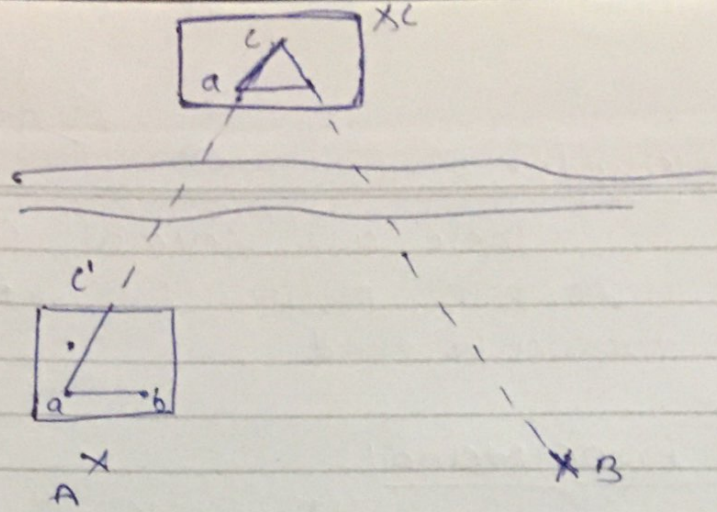


Resection is the process of determining location of plane table st^n [C] with reference to ground st^n s [A & B] which are already plotted on the drawing sheet as point a & b

If the orientation of plane table at st^n C is correct then intersection of two rays from a & b will give correct location of C therefore now problem reduces to obtaining correct orientation at C

1) Back Ray Method:

This method can be used only if plane table st^n C was already selected and a ray has been drawn



from one of the st^n towards $st^n c$ but exact position is not marked on the drawing sheet

Two point problem:-

The two point problem is used to locate the position of plane table $st^n c$ on the drawing sheet by observing two well defined points (say A & B) whose position is already marked on the drawing sheet as point a & b respectively.

Steps:-

Three point problem:-

Three point problem consist of locating plane table station (P) on the drawing sheet with the help of 3 well defined ground st^n s (A, B & C) which are already marked on the drawing sheet as point a, b & c respectively.

Three point problem can be solved by several methods

- 1) Graphical method
- 2) Trial & Error method
- 3) Mechanical method
- 4) analytical method
- 5) Geometrical construction method.

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1) Graphical Method:-

There are several Graphical Methods for three point problem but Bessel's Solⁿ is most commonly used.

2) Trial and Error Method:-

This method is very commonly used in field as it gives very accurate result and less time consuming.

Lehmann's Rule are extremely useful while taking the trials. Hence this method is also called as Lehmen's Method.

In Trial & Error method orientation is done by making a Number of Trials till correct orientation is achieved.